Arithmetic geometry of toric varieties

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There is a very rich theory linking the algebraic geometry of toric varieties with combinatorial properties. For instance, to a toric variety X provided with an ample divisor D we can associate a lattice polytope Δ. We can recover the variety and the divisor from the polytope and many properties of (X, D) can be read from this polytope. For instance the degree of D is given by $n!$ times the volume of the polytope ($n = \dim(X)$) and a basis of the global sections of $\mathcal{O}(D)$ is given by the integral points of the polytope. In a join project with P. Philippon and M. Sombra we have extended this toric dictionary to the Arakelov theory of toric varieties. Each toric variety has a canonical model over $\mathbb{Z}$. To a semipositive hermitian metric on $\mathcal{O}(D)$, invariant under the action of the compact torus, we associate a concave function ϑ on Δ. Called the roof function. The objective of this talk is to convince you that the roof function can be seen as an extended polytope that codifies most of the Arakelovian properties of X. For instance we can compute from it the height of X, the arithmetic volume of X, the essential and absolute minima and whether there is equidistribution of Galois orbits of small points.

Joint work with Martin Sombra (ICREA and Universitat de Barcelona), Patrice Philippon (CNRS and Institut de Mathématiques de Jussieu), Atsushi Moriwaki (University of Kyoto) and Juan Rivera-Letelier (University of Rochester).