Cluster Theory

Gordana Todorov
Northeastern University, United States
g.todorov@neu.edu

Cluster algebras were introduced by Fomin and Zelevinsky in 2000 in the context of Lie theory to deal with the total positivity and Lustig's dual canonical basis. Cluster algebras are commutative algebras generated by cluster variables which are obtained in a very particular way, using mutations guided by sequences of skew symmetrizable matrices, which are also mutated in the process.

After the introduction of cluster algebras, there was a large amount of mathematics developed connecting cluster algebras to many fields of mathematics: representation theory of finite dimensional algebras, Auslander-Reiten theory, combinatorics, Poisson geometry, Teichmuller theory, tropical geometry, integrable systems and more.

Already at the early stages, additive categorification was introduced for acyclic cluster algebras, i.e. for those cluster algebras which correspond to the quivers with no oriented cycles. Cluster categories were defined as certain orbit categories of the derived categories of the categories of quiver representations. It was shown that there is a beautiful correspondence between the fundamental notions of cluster algebras: cluster variables, clusters, cluster mutations and, the notions in the associated cluster category: indecomposable rigid objects, cluster tilting objects and tilting mutations.

Since the original motivation for the introduction of cluster categories was giving categorical interpretation to the combinatorics of the cluster algebras, in this talk I will mostly concentrate on this relation between cluster algebras and cluster categories.