Hopf-Lie theory on hyperplane arrangements

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The braid hyperplane arrangement plays a central role in the study of various important notions in general algebra. This role may not be immediately apparent, but when it is made explicit, one discovers that these notions can be extended to a new setting in which an arbitrary real hyperplane arrangement takes center stage. This results in a new theory with strong connections with geometric combinatorics, semigroup theory and other areas of classical algebra. This theory has been the focus of my attention for the past few years and I have been working on it in close collaboration with Swapneel Mahajan. I will start by discussing a few basic notions pertaining to real hyperplane arrangements, focusing on the Tits product of faces. Then I will try to support the central claim by defining extensions of the notion of Hopf algebra, Lie algebra, and operads. I will mention extensions of a few selected results such as a theorem of Joyal, Klyachko and Stanley (relating the free Lie algebra to the partition lattice), the Cartier-Milnor-Moore theorem (relating Hopf and Lie algebras), and concepts such as Koszul duality. The conclusion is that much of this classical theory admits an extension relative to a real hyperplane arrangement.

Joint work with Swapneel Mahajan (Indian Institute of Technology Mumbay, India).