On computational aspects of the generalized differential Lüroth's theorem

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Let $\mathcal{F}$ be a differential field of characteristic 0, $\mathbf{t} = t_1, \ldots, t_m$ a set of differential indeterminates over $\mathcal{F}$, and $\mathcal{F}\langle\mathbf{t}\rangle$ the field of differential rational functions. The generalized differential Lüroth's theorem proposed by Kolchin states that, for every differential subfield $\mathcal{G}$ of $\mathcal{F}\langle\mathbf{t}\rangle$ such that the extension $\mathcal{G}/\mathcal{F}$ has differential transcendence degree 1, there exists $v \in \mathcal{F}\langle\mathbf{t}\rangle$ with $\mathcal{G} = \mathcal{F}\langle v\rangle$. This result generalizes the differential Lüroth theorem proved by Ritt for $m = 1$.

We will discuss effectivity issues of the generalized differential Lüroth theorem. If $\mathcal{G}$ is generated by a finite family of differential rational functions in $\mathcal{F}\langle\mathbf{t}\rangle$ of bounded orders and degrees, we will present upper bounds for the order and the degree of any Lüroth generator v of $\mathcal{G}$ over $\mathcal{F}$. These are the first known bounds for arbitrary m and, in the case $m = 1$, they improve the previous degree bounds. In addition, we will show that a Lüroth generator can be computed by means of classical techniques from computer algebra applied to a polynomial ideal associated with the given generators. Finally, we will show how to determine whether a given differentially finitely generated subfield of $\mathcal{F}\langle\mathbf{t}\rangle$ has differential transcendence degree 1 over $\mathcal{F}$.

Joint work with Lisi D'Alfonso (Universidad de Buenos Aires, Argentina) and Pablo Solernó (Universidad de Buenos Aires - CONICET, Argentina).