Tannaka duality for algebraic groups

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The Chevalley's structure theorem states that any connected algebraic group over an algebraically closed field is the extension of an abelian variety by a connected affine algebraic group. In view of this result, the theory of algebraic groups has been developed along two directions: the study of linear (affine) algebraic groups and that of abelian varieties. The representation theory of affine algebraic groups plays an important role in their study: the (classical) Tannaka duality theorem guarantees that an affine algebraic group can be recovered from its category or representations.

In this talk we propose a representation theory for arbitrary algebraic groups, as follows: let G be an algebraic group G consider its Chevalley decomposition $1 \to G_{aff} \to G \to A \to 0$. A representation of G is a homogeneous vector bundle $E \to A$ together with regular action $\varphi : G \times E \to E$, linear on the fibres and such that the induced morphism $\widetilde{\varphi} : A \times A \to A$ is the product in A (recall that $A \times A$ is the Albanese variety of $G \times E$). We will define the category of representations of G, and prove that a generalisation of Tannaka duality theorem is valid in this context, therefore allowing us to recover an algebraic group from its category of representations.

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