Intersections of Amoebas

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Given an Laurent polynomial $f \in \mathbb{C}[z_1^{\pm 1}, \ldots, z_n^{\pm 1}]$ the amoeba $\mathcal{A}(f)$ (introduced by Gelfand, Kapranov, and Zelevinsky '94) is the image of its variety $\mathcal{V}(f) \subseteq (\mathbb{C}^*)^n = (\mathbb{C} \setminus \{0\})^n$ under the $\mathrm{Log}|\cdot|$-map

$$\mathrm{Log}|\cdot| : (\mathbb{C}^*)^n \to \mathbb{R}^n, \quad (z_1, \ldots, z_n) \mapsto (\log|z_1|, \ldots, \log|z_n|).$$

Amoebas have amazing structural properties; they are related to various mathematical subjects like complex analysis, the topology of real algebraic curves, nonnegativity of polynomials, dynamical systems, and particularly tropical geometry.

While amoebas of hypersurfaces have been studied intensively during the last years, the non-hypersurface case is not understood so far. Here, we investigate intersections of amoebas of n hypersurfaces in $(\mathbb{C}^*)^n$, which are canonical supersets of amoebas given by non-hypersurface varieties. As a main result we present an amoeba analog of the classical Bernstein Theorem from combinatorial algebraic geometry providing an upper bound for the number of connected components of such intersections.

We also show how the order map for hypersurface amoebas can be generalized in a natural way to intersections of amoebas. Particularly, analogous to the case of amoebas of hypersurfaces, the restriction of this generalized order map to a single connected component of the intersection is still 1-to-1.

For further information see http://arxiv.org/abs/1510.08416.

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