Rational Harnack Curves on Toric Surfaces

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Harnack curves are a family of real algebraic curves who are distinguished because their topology is well understood, meaning that Hilbert's 16th problem is solved for these curves. Let f be a 2 variable real polynomial whose Newton polygon is Δ and let C be the curve defined as the zeros of f inside the toric variety X_Δ. The original definition of Harnack curves by Mikhalkin states that the real part of C, $\mathbb{R}C \subseteq \mathbb{R}X_\Delta$, is a Harnack curve if and only if the following conditions are satisfied:

1. The number of connected components of $\mathbb{R}C$ is maximal, that is $g+1$, where g is the arithmethic genus of C.

2. Only one component O intersects the axes of $\mathbb{R}X_\Delta$.

3. Let $l_1, \ldots, l_n$ be the axes of X_Δ ordered in a way such that it agrees with the cyclical order of their corresponding sides of Δ and let $d_1, \ldots, d_n$ be the integer lengths of the corresponding sides. Then O can be divided into disjoint arcs $\alpha_1, \ldots \alpha_n$ such that $\alpha_i \cap l_i = d_i$ and $\alpha_i \cap l_j = 0$ when $j \neq i$.

These curves have several different characterizations, for example, its amoeba (the image of C under the map $(z,w) \mapsto (\log|z|, \log|w|)$) is of maximal area. These curves have applications to physics through dimer theory. In this poster we focus on rational Harnack curves, which are Harnack curves of genus 0 and we show how these curves can be explicitly parametrized using the homogeneous coordinates of X_Δ.

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