Shapes of the simplest minimal free resolutions in $\mathbb{P}^1 \times \mathbb{P}^1$

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The study of graduate syzygies for the standard case 3 homogeneous polynomials is now well known, but the general context (especially in the multihomogeneous case) is of greater interest but involves further difficulties and is very much unknown.

In 2007, Cox, Dickenstein and Schenck have analysed in depth, from both a geometric and an algebraic perspective, the minimal free resolution of an ideal given by 3 bihomogeneous polynomials of bidegree $d = (2, 1)$ in $R = k[X_1, X_2][X_3, X_4]$. In particular, an accurate description of the non-Koszul syzygies is obtained from an application of the Künneth formula and the Serre duality for $\mathbb{P}^1$; see the 1994 paper by Weyman and Zelevinsky on the subject.

In this talk, our goal is to give a detailed description of the (multi)graded minimal free resolution of an ideal I of R, generated by 3 bihomogeneous polynomials defined by $\mathbf{f} = (f_1, f_2, f_3)$ with bidegree (d_1, d_2), $d_i > 0$ and such that $V(I)$ is empty in $\mathbb{P}^1 \times \mathbb{P}^1$. We will precise the shape of the resolution in degree $d = (1, n)$, and explain how non-genericity (factorization) of the f_i's determine the resolution.

Joint work with Alicia Dickenstein (Universidad de Buenos Aires, Argentina) and Hal Schenck (University of Illinois, USA).