On Hopf orders and Kaplansky's sixth conjecture

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A theorem of Frobenius states that the degree of any complex irreducible representation of a finite group G divides the order of G. This is proved using the following specific property of the group algebra $\mathbb{C}G$: it is defined over $\mathbb{Z}$ or, in other words, the group ring $\mathbb{Z}G$ is a Hopf order of $\mathbb{C}G$.

Kaplansky's sixth conjecture predicts that Frobenius Theorem holds for complex semisimple Hopf algebras. There are several partial results in the affirmative. Compared to the case of groups, the main difficulty to prove this conjecture (if true) is that it is not guaranteed that a complex semisimple Hopf algebra H is defined over $\mathbb{Z}$ or, more generally, over a number ring. If it would be so, Larson proved that H satisfies Kaplansky's sixth conjecture. The question whether every complex semisimple Hopf algebra can be defined over a number ring has always been behind this conjecture.

In this talk we will answer this question in the negative. The family of examples that we will handle, constructed by Galindo and Natale, are Drinfeld twists of certain group algebras. The key fact is that the twist contains a scalar fraction, which makes impossible to define such Hopf algebras over a number ring.

The results that will be presented are part of a joint work with Ehud Meir (University of Hamburg) published in Trans. Amer. Math. Soc. and available at arXiv.org.