A study on clean rings

Laiz Valim da Rocha
Universidade Federal Fluminense, Brasil
laizvalim@gmail.com

A ring is said to be clean if every element can be written as sum of a unit and an idempotent. These rings were defined by Nicholson [5], while studying exchange rings. The class of clean rings is located among other well known classes of rings [3]. In the realm of group rings, these properties have been studied from 2001 [2] on with the aim of characterizing the rings R and groups G such that the group ring RG is clean.

The study of $*$-clean rings was motivated by a question made by T. Y. Lam at the Conference on Algebra and Its Applications, in March 2005, at the Ohio University: which von Neumann algebras are clean as rings? Since von Neumann algebras are $*$-rings (i.e., rings with an involution), it is more natural to work with projections (idempotents that are symmetric under the ring involution) than with idempotents.

So, in 2010 Vaš defined $*$-clean rings [6]: a $*$-ring in which every element may be written as a sum of a unit and a projection. Clearly, every $*$-clean ring is a $*$-ring and is a clean ring.

Every group G is endowed with the classical involution $g \mapsto g^{-1}$. If R is a commutative ring, for instance, the R-linear extension of the classical involution in G is the classical involution in RG. $*$-clean group rings were first studied in 2011 [4]. However very little is still known about when a group ring is $*$-clean (not even the case of the group ring RG, where R is a commutative ring and G is a cyclic group, is fully stablished!).

In this talk, we present clean rings, their relationship with other types of rings [3] and some recent results [1]. Let R be a commutative local ring. I will provide necessary and sufficient conditions for the group rings RC_3 and RC_4 to be $*$-clean, where C_n denote the cyclic group with n elements.

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