Semiclean Rings

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A ring R with unity is said to be clean if every element in the ring can be written as the sum of a unit and an idempotent of the ring. These rings were introduced by Nicholson, [?], in his study of lifting idempotents and exchange rings. The division rings, boolean rings and local rings are examples of clean rings.

In the article [?], a new class of rings is defined; semiclean rings. A ring R with unity is called semiclean if, every $x \in R$, x = u + a with $u \in \mathcal{U}(R)$ where a is periodic element, i.e., $a^k = a^l$ with $k, z \in \mathbb{Z}$ and $k \neq z$. Therefore, every semiclean ring is a clean ring, because the idempotents elements of ring are periodics. Nicholson e Han, [?], demonstrated that group ring $Z_{(7)}C_3$ is not a clean ring. Yuanqing Ye showed, in the article [?], that the group ring $Z_{(p)}C_3$ is an semiclean ring. This result assures that the two classes, clean and semiclean, are different.

Motivated by the article [?], we intend to investigate if the Yuanging Ye's demonstration can be generalized, as in the cases $Z_{(11)}C_5$ and $Z_{(p)}C_5$, in search of a possible answer about the ring $Z_{(p)}C_q$ with p and q relatively primers.

Joint work with Elen Deise Assis Barbosa(Universidade Federal da Bahia, Brasil).