The Normalizer Property and its Relation with Extensions of Groups

Jacqueline C. Cintra
UFBA - Universidade Federal da Bahia, Brazil
jcintra@ufba.br

The determination of the normalizer of the basis group in the group of units of the associated group ring is a question that naturally imposes by itself. In integral group rings, in particular, it has been observed that, for important classes of finite groups, this normalizer is minimal, in other words, $\mathcal{N}_{\mathcal{U}}(G) = G \cdot Z$. When this occurs, we say that the group in question and its integral group ring satisfy the normalizer property. This property, also known as (Nor), has recently gained great importance when Mazur, in [Ma95], noticed an interesting relation with the famous problem of isomorphism in integral group rings also known as (Iso). Exploring this connection, Hertweck in [He01] found an example of a finite group that does not satisfy (Nor), and indirectly, by the relation mentioned above, obtained a counterexample to (Iso). Given that the counter example of Hertweck to (Nor) consists of an extension given by a semidirect product, but [LPS99] proves that extensions given by direct products are solutions (Nor), it is important to investigate which other other extensions of finite groups answer the property. Recently, Petit Lobão e Sehgal in [PeS03] demonstrated the validity of (Nor) for the class of complete monomial groups; in other words, a wreath extension of a finite nilpotent group with the symmetric group on m letters. Zhengxing Li e Jinke Hai in a series of articles, among which we have [HL12], [HL12b], HL11], also obtained interesting solutions of this property. The purpose of this work is to verify the relation between (Nor) and extensions of groups, where such component groups are solutions (Nor), in order to obtain necessary and sufficient conditions to find positive solutions to the property in question.

References

[HL12] HAI, J.; Li, Z. The Normalizer Property for Integral Group Rings of Some Finite Nilpotent-by-Nilpotent Groups, *Communications in Algebra*, v. 40, n. 7, p. 2613-2627, 2012.

[HL12b] HAI, J.; LI, Z.The Normalizer Property for Integral Group Rings of Finite Solvable T-Groups, *Journal of Group Theory*, v. 15, n. 2, p. 237-243, 2012.

[HL11] HAI, J.; LI, Z. The normalizer property for integral group rings of wreath products of finite nilpotent groups by some 2-groups, *Journal of Group Theory*, v. 14, n. 2, p. 299-306, 2011.

[He01] HERTWECK, M. A counterexample to the isomorphism problem for integral group rings, *Annals of Mathematics*, v. 154, n. 1, p. 115-138, 2001.

[LPS99] LI, Y.; PARMENTER, M. M.; SEHGAL, S. K. On the normalizer property for integral group rings. *Communications in Algebra*, v. 27, n. 9, p. 4217-4223, 1999.

[Ma95] MAZUR, M. On the isomorphism problem for integral group rings of infinite groups, *Expositiones Mathematicae*, v. 13, n. 5, p. 433-445, 1995.

[PeS03] PETIT LOBÃO, T.; SEHGAL, S. K. The Normalizer Property for Integral Group Rings of Complete Monomial Groups, *Communications in Algebra*, v. 31, n. 6, p. 2971-2983, 2003.

Joint work with Thierry Petit Lobão (Universidade Federal da Bahia).