$*$ - Clean Group Algebras

Gianfranco Osmar Manrique Portuguez
Universidade Federal Fluminense, Brasil
gian1427@gmail.com

An element of a(n associative) ring (with 1) is clean if it is the sum of a unit and an idempotent. A ring is clean if every element in it is clean. The property of cleanness was formulated by Nicholson [4] in the course of his study of exchange rings. From then on, several related concepts were proposed: uniquely clean rings, strongly clean rings, weakly clean rings, $*$ - clean rings, r - clean rings, nil - clean rings, to cite a few. In the realm of group rings, these properties have been studied from 2001 [2] on with the aim of characterizing the rings R and groups G such that the group ring RG is clean.

In 2010 Vas proposed the definition of a $*$ - clean ring ("star"- clean) [5]: a $*$ - ring (i.e., rings with an involution) in which every element may be written as a sum of a unit and a projection. Clearly, every $*$-clean ring is a *star* - ring and is a clean ring. In [5], Vas asked: when is a $*$ - ring clean, but not $*$-clean? Every group G having an element g $\neq 1$, with $|\langle g\rangle| \neq 2$, is endowed with the classical involution $g \mapsto g^{-1}$. Because of that, group rings RG are almost always $*$ - rings: if R is a commutative rings, for instance, an involution in RG is obtained from the R - linear extension of the classical involution in G (and is also called the classical involution in RG). The $*$-cleanness of group rings was first approached in 2011 [3]. Even though some instances of group rings are answers to Vas's question [1], very little is still known about conditions under which a group ring with the classical involution is $*$-clean (not even the case of the group ring RG, where R is a commutative ring and G is a cyclic group, is fully stablished!).

In this talk, I present some recent results [1]. Let R be a commutative local ring. I will present RS_3 as an answer to Vas 's question, and I will provide necessary and sufficient conditions for the group ring RQ_8 to ber $*$ - clean, where Q_8 is the quaternion group of 8 elements.

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