Partial actions and subshifts

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An arbitrary (one-sided) subshift X over a finite alphabet Λ with n letters can be naturally endowed with a partial action θ of the free group $\mathbb{F}_n$ with n free generators $g_\lambda, (\lambda \in \Lambda)$, such that g_λ maps x to λx, where x is an element in X for which $\lambda x \in X$. Naturally g_λ^{-1} removes λ from λx. We call θ the standard partial action, and it is a starting point to constract a C^*-algebra $\mathcal{O}_X^*$ associated with X, as well as an abstract algbera $\mathcal{O}_X^K$ over an arbitrary field K of characteristic 0. Both $\mathcal{O}_X^K$ and $\mathcal{O}_X^*$ are defined in a fairly similar way: using the standard partial action we construct a partial representation u of $\mathbb{F}_n$ into an appropriate algebra (which depends on whether the case is abstract or C^*) and then define $\mathcal{O}_X^K$ (or $\mathcal{O}_X^*$) as the subalgebra (respectively, a C^*-subalgebra) generated by $u(\mathbb{F}_n)$. Then using a general procedure (see [4,Proposition 10.1] we obtain a partial action τ of $\mathbb{F}_n$ on a commutative subalgebra $\mathcal{A}$ and prove that $\mathcal{O}_X^K$ is isomorphic to the crossed product $\mathcal{A} \rtimes_\tau \mathbb{F}_n$. In the C^* case (see [3,Theorem 9.5]), due to an amenability property, $\mathcal{O}_X^*$ is isomorphic to both the full and the reduced crossed product: $\mathcal{O}_X^* \cong \mathcal{D} \rtimes_\tau \mathbb{F}_n \cong \mathcal{D} \rtimes_\tau^{\rm red} \mathbb{F}_n$, where $\mathcal{D}$ is a commutative C^*-algebra defined in a similar way as $\mathcal{A}$. This gives a possibility to study algebras related to subshifts using crossed products by partial actions. It turns out that $\mathcal{O}_X^*$ is isomorphic to the C^*-algebra defined by T. M. Carlsen in [1] in a somewhat different way (see [3,Theorem 10.2]). In particular, if X is a Markov subshift, then $\mathcal{O}_X^*$ is isomorphic to the Cuntz-Krieger algebra defined in [2]. The C^* version is elaborated in the preprint [3], in which, amongst several related results, a criterion is given for simplicity of $\mathcal{O}_X^*$ (see [3,Theorem 14.5]).

[1] T. M. Carlsen, Cuntz-Pimsner, C^*-algebras associated with subshifts, Internat. J. Math., 19 (2008), 47–70.

[2] J. Cuntz, W. Krieger, A class of C^*-algebras and topological Markov chains, Invent. Math., 63 (1981), 25–40.

[3] M. Dokuchaev, R. Exel, Partial actions and subshifts, Preprint, arXiv:1511.00939v1 (2015).

[4] R. Exel, Partial Dynamical Systems, Fell Bundles and Application, to be published in a forthcoming NYJM book series. Available from http://mtm.ufsc.br/?exel/papers/pdynsysfellbun.pdf.

Joint work with Ruy Exel (Universidade Federal de Santa Catarina, Brazil).