On trees with the same restriction of the chromatic symmetric function and solutions to the Prouhet-Tarry-Escott problem

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On the one hand, the Prouhet-Tarry-Escott problem asks, given k be a positive integer, whether there exist integer sequences $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$, distinct up to permutation, such that

$$a_1^\ell + \ldots + a_n^\ell = b_1^\ell + \ldots + b_n^\ell \quad \text{for all } 1 \leq \ell \leq k.$$

This is an old problem in number theory (Prouhet 1851), and solutions are known to exist for every k.

On the other hand, the chromatic symmetric function was introduced by Stanley in 1995 as a symmetric function generalization of the chromatic polynomial of a graph. It is an open problem to know whether there exist non-isomorphic trees with the same chromatic symmetric function.

In this talk, we show how to encode solutions of the Prouhet-Tarry-Escott problem as non-isomorphic trees having the same restriction of the chromatic symmetric function. As a corollary, we find a new class of trees that are distinguished by the chromatic symmetric function up to isomorphism.

Joint work with Anna de Mier (Universidad Politécnica de Cataluña, España) and José Zamora (Universidad Andres Bello, Chile).