A proof of the peak polynomial positivity conjecture

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Given a permutation $\pi = \pi_1\pi_2\cdots\pi_n \in \mathfrak{S}_n$, we say an index i is a peak if $\pi_{i-1} < \pi_i > \pi_{i+1}$. Let $P(\pi)$ denote the set of peaks of π. Given any set S of positive integers, define $P_S(n) = \{\pi \in \mathfrak{S}_n : P(\pi) = S\}$. In 2013 Billey, Burdzy, and Sagan showed that for all fixed subsets of positive integers S and sufficiently large n, $|P_S(n)| = p_S(n)2^{n-|S|-1}$ for some polynomial $p_S(x)$ depending on S. They gave a recursive formula for $p_S(n)$ involving an alternating sum, and they conjectured that the coefficients of $p_S(x)$ expanded in a binomial coefficient basis centered at $\max(S)$ are all nonnegative. In this talk we will share a different recursive formula for $p_S(n)$ without alternating sums, and we use this recursion to prove that their conjecture is true.

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