Similarity between the algebraic structure associated with projective space and combinatorial design via Hasse diagram

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Combinatorial design is an important combinatorial structure having a high degree of regularity and which is related to the existence and construction of systems of sets with finite cardinality, [1]. As examples we mention the existing relationship between error-correcting codes in the Hamming space and combinatorial design, where the codewords of weight 3 of the Hamming code form a triple Steiner system STS(7), a projective plane of order 2, known as the Fano plane, [2], as well as q-analogs of a code whose codewords have constant Hamming weight in the Hamming space, a code belonging to a Grassmannian in the projective space, [3,4]. Projective space of order m over a finite field $\mathbb{F}_p$, denoted by $\mathcal{P}(\mathbb{F}_p^m)$, (note that $\mathbb{F}_{p^m}$ is isomorphic to $\mathbb{F}_p^m$), is the set of all the subspaces in the vector space $\mathbb{F}_p^m$. The projective space endowed with the subspace distance $d(X,Y) = dim(X) + dim(Y) - 2dim(X \cap Y)$ is a metric space. Hence, the subspace code $\mathcal{C}$ with parameters (n, M, d) in the projective space is a subset of $\mathcal{P}(\mathbb{F}_p^m)$ with cardinality M with a subspace distance at least d between any two codewords, [5]. In this paper we show the existing similarity between the Hasse diagram of an Abelian group consisting of the product of multiplicative finite Abelian groups $\mathbb{Z}_p^m$ and the Hasse diagram of the projective space $\mathcal{P}(\mathbb{F}_p^m)$, with the aim to provide the elements that may be useful in the identification and in the construction of good subspaces codes, [6].

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