Some remarks on the asymptotic behavior of cyclic AG-codes

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It was proved in [St06] and [B06] that several classes of algebraic geometry codes, such as transitive codes, self-dual codes and quasi transitive codes among others, are asymptotically good over finite fields with square and cubic cardinality. Similar results were proved in [BBGS14] for general non-prime fields. In fact, some of them attain the well known Tsfasman-Vladut-Zink bound and also improvements for another well known bound of Gilbert-Varshamov were given. These results were achieved by considering algebraic geometry codes associated to asymptotically good towers of function fields over suitable finite fields.

Remarkably few things are known, so far, with regard to the asymptotic behavior of the class of cyclic codes. Perhaps the most interesting result in this direction is the one due to Castagnoli who proved in [Ca89] that the class of cyclic codes whose block lengths have prime factors belonging to a fixed finite set of prime numbers is asymptotically bad. This result implies that the construction of cyclic AG-codes in the standard way, would lead to a sequence of codes asymptotically bad.

In this talk we will discuss how different the situation is when dealing with the asymptotic behavior of transitive (or quasi transitive) AG-codes and cyclic AG-codes, which are particular cases of transitive AG-codes, [CPT16]. We will conclude that towers with only totally ramified places in the tower, which are nice candidates for good asymptotic behavior, have to be discarded for the construction of potentially good sequences of cyclic AG-codes, if we want to use all the techniques and results that were successful in the transitive case. All of this, together with Castagnoli's result, provide some good reasons to think that towers of function fields may not be adequate to address the problem of the asymptotic behavior of cyclic codes, as long as the sequence of cyclic AG-codes is constructed using automorphisms of the function fields in the tower. It is clear that the design of new methods to produce cyclic AG-codes is an interesting and challenging problem with potential consequences in the study of the asymptotic behavior of cyclic codes.

References

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