A problem of Beelen, Garcia and Stichtenoth on an Artin-Schreier tower

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A *tower of function fields* over $\mathbb{F}_q$ is a sequence of algebraic function fields $\mathcal{F} = \{F_i\}_{i=0}^{\infty}$ such that for all $i \geq 0$ $F_i \subsetneq F_{i+1}$, F_{i+1}/F_i is a separable finite extension, $\mathbb{F}_q$ is algebraically closed in F_i and there exists F_j with genus greater than one.

A tower $\mathcal{F}$ is called *asymptotically good* if $\gamma(\mathcal{F}) < \infty$ and $\nu(\mathcal{F}) > 0$ where

$$\gamma(\mathcal{F}) := \lim_{i\to\infty} g(F_i)/[F_i : F_0] \quad \text{and} \quad \nu(\mathcal{F}) := \lim_{i\to\infty} N(F_i)/[F_i : F_0],$$

$g(F_i)$ is the genus of F_i and $N(F_i)$ is the number of rational places of F_i. Otherwise, $\mathcal{F}$ is called *asymptotically bad.*

In 2006 Beleen, Garcia and Stichtenoth proved that any recursive tower of function fields over $\mathbb{F}_2$ defined by $g(Y) = f(X)$ with $g(T), f(T) \in \mathbb{F}_2(T)$ and $\deg f = \deg g = 2$ is defined by the Artin-Schreier equation

$$Y^2 + Y = \frac{1}{(1/X)^2 + (1/X) + b} + c, \tag{1}$$

with $b, c \in \mathbb{F}_2$. They checked that all the posible cases were already considered in previous works, except when $b = c = 1$. In fact, they left as an open problem to determine whether or not this tower is asymptotically good over $\mathbb{F}_{2^s}$ for some positive integer s.

In this talk we will show that the recursive tower defined by equation (1) with $b = c = 1$ is asymptotically bad over $\mathbb{F}_{2^s}$ when s is odd and where the main difficulty arises in the study of this tower when s is even.

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