New constructions of algebraic lattices

Antonio Aparecido de Andrade
São Paulo State University at São José do Rio Preto, Brasil
andrade@ibilce.unesp.br

In algebraic number theory, it is better known the ring of integers of the cyclotomic field and the ring of integers of their maximal real subfield. An important result of this area, the Kronecker-Weber Theorem, states that every abelian number field is contained in a cyclotomic field. Thinking about this, we can ask ourselves what is the ring of integers of each abelian number field and if this ring of integers has a power basis, this is, if the ring of integers is generated by an element over Z. In this line, to construct lattices in odd dimensions, we can not use cyclotomic fields, but we can use their subfields. Also, the maximal real cyclotomic subfields are not suficient to solve the problem of find algebraic lattices that has better center density. Trying to solve this problem mainly in odd dimensions, we are using abelian number fields. For this task we need the ring of integers of abelian number fields, which is presented by the Leopoldt's Theorem (1959) or its version given by Lettl (1990). In this work, we intend to present the Leopoldt's Theorem in the version of Lettl and elucidate why it can be useful to construct algebraic lattices with better center desnsity.

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Joint work with Robson Ricardo de Araujo. Department of Mathematics, State University of Campinas, Campinas - SP, Brasil.