On the Densest Lattices from Number Fields and Division Algebras

Carina Alves
São Paulo State Universtiy - UNESP - Rio Claro, Brasil
carina@rc.unesp.br

Algebraic lattices are lattices obtained via the ring of integers, $\mathcal{O}_{\mathbb{F}}$, of a number field $\mathbb{F}$. They can be constructed considering geometric representations of integral ideals in $\mathcal{O}_{\mathbb{F}}$. This latter method was successfully used by Craig to construct the Leech lattice from a properly chosen integral ideal $\mathcal{I}$ in $\mathbb{Z}[\zeta_{39}]$, the ring of integers of the cyclotomic field $\mathbb{F} = \mathbb{Q}(\zeta_{39})$. In addition to the Leech lattice, Craig showed that the lattices D_4, E_8, K_{12}, and Λ_{16} can all be obtained from properly chosen integral ideals in rings of cyclotomic integers. Bayer-Fluckiger showed that E_8 can be obtained via a ideal $\mathcal{I}$ in $\mathcal{O}_{\mathbb{F}}$, $\mathbb{F} = \mathbb{Q}(\zeta_{15})$, $\mathbb{Q}(\zeta_{20})$, $\mathbb{Q}(\zeta_{24})$. Versions of dense lattices are of interest from the practical viewpoint as they are suitable for data transmission. More recently, the need for higher data transmission has led to consider communication channels using multiple antennas at both transmitter and receiver ends (MIMO). In the case of space-time codes, it is natural to consider a lattice from an ideal of a maximal order of the division algebra. Codewords are usually (in narrow band systems) built over the complex field. However for ultra wideband communication, one needs to design them over the real field. Thus, having the construction of lattices as our goal, in this work we present constructions of dense lattices from maximal orders of the division algebras over a totally real number field.

Joint work with Sueli I.R. Costa (Institute of Mathematics, University of Campinas, Campinas-SP, Brazil, sueli@ime.unicamp.br). and Cintya W. O. Benedito (Institute of Mathematics, University of Campinas, Campinas-SP, Brazil, cwinktc@hotmail.com).