On hemi-implicative semilattices

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In this talk we introduce and study classes of algebras that properly include varieties of interest for logic. These algebras are obtained by weakening the main features of Heyting algebras but retaining most of their algebraic consequences. To be more precise, we give the following definition.

Definition: An algebra $(H, \wedge, \rightarrow, 1)$ of type $(2, 2, 0)$ is a *hemi-implicative semilattice* if the following conditions hold:

$H1$: $(H, \wedge, 1)$ is a bounded semilattice.

$H2$: For every $a, b, c \in H$, if $c \leq a \rightarrow b$ then $a \wedge c \leq b$.

$a \rightarrow a = 1$ for every $a \in H$.

An algebra $(H, \wedge, \vee, \rightarrow, 0, 1)$ of type $(2, 2, 2, 0, 0)$ is said to be a *hemi-implicative lattice* if $(H, \wedge, \vee, 0, 1)$ is a bounded distributive lattice and $(H, \wedge, \rightarrow, 1)$ is a hemi-implicative semilattice.

If $(H, \wedge)$ is a semilattice with a binary operation $\rightarrow$, then H satisfies the condition (H2) if and only if it holds the inequality $a \wedge (a \rightarrow b) \leq b$ for every $a, b \in H$. Thus, the condition (H2) is a kind of modus ponens rule. Moreover, the class of hemi-implicative semilattices is a variety and the class of hemi-implicative lattices is also a variety.

Implicative semilattices introduced by Nemitz are examples of hemi-implicative semilattices. Some examples of hemi-implicative lattices are the semi-Heyting algebras, which were introduced by H.P. Sankappanavar in as an abstraction of Heyting algebras, and the RWH-algebras, which were introduced by Celani and Jansana in as another possible generalization of Heyting algebras.