An optimal axiomatization of the set of central elements

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We say that a variety $\mathcal{V}$ with $\vec{0}$ and $\vec{1}$ has "definable factor congruences" if there exists a first-order formula defining every factor congruence in every algebra $\mathbf{A} \in \mathcal{V}$ in terms of its associated *central elements*. When there is a $(\bigwedge p = q)$-formula satisfying this condition we say that $\mathcal{V}$ has "equationally definable factor congruences". We denote by $Z(\mathbf{A})$ the set of central elements of $\mathbf{A}$. In "Varieties with equationally definable factor congruences II" we give an axiomatization of $Z(\mathbf{A})$ for varieties with equationally definable factor congruences which is optimal in the sense of its quantificational complexity. The given axiomatization is not a set of positive formulas nor a set of Horn formulas. There are several examples which show that in the general case, varieties with equationally definable factor congruences do not admit an axiomatization of $Z(\mathbf{A})$ by a set of positive formulas. However, as we will see, there is an axiomatization of $Z(\mathbf{A})$ which is a set of Horn formulas with the optimal quatificational complexity, which evidences the already known fact that central elements are preserved by direct products.

Joint work with Diego Vaggione (Universidad Nacional de Córdoba).