XXI CLA - Plenary talks

Plenary talk - July 25, 10:00 – 10:50

Cluster Theory

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Cluster algebras were introduced by Fomin and Zelevinsky in 2000 in the context of Lie theory to deal with the total positivity and Lustig's dual canonical basis. Cluster algebras are commutative algebras generated by cluster variables which are obtained in a very particular way, using mutations guided by sequences of skew symmetrizable matrices, which are also mutated in the process.

After the introduction of cluster algebras, there was a large amount of mathematics developed connecting cluster algebras to many fields of mathematics: representation theory of finite dimensional algebras, Auslander-Reiten theory, combinatorics, Poisson geometry, Teichmuller theory, tropical geometry, integrable systems and more.

Already at the early stages, additive categorification was introduced for acyclic cluster algebras, i.e. for those cluster algebras which correspond to the quivers with no oriented cycles. Cluster categories were defined as certain orbit categories of the derived categories of the categories of quiver representations. It was shown that there is a beautiful correspondence between the fundamental notions of cluster algebras: cluster variables, clusters, cluster mutations and, the notions in the associated cluster category: indecomposable rigid objects, cluster tilting objects and tilting mutations.

Since the original motivation for the introduction of cluster categories was giving categorical interpretation to the combinatorics of the cluster algebras, in this talk I will mostly concentrate on this relation between cluster algebras and cluster categories.

Plenary talk - July 25, 11:30 – 12:20

Hopf-Lie theory on hyperplane arrangements

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The braid hyperplane arrangement plays a central role in the study of various important notions in general algebra. This role may not be immediately apparent, but when it is made explicit, one discovers that these notions can be extended to a new setting in which an arbitrary real hyperplane arrangement takes center stage. This results in a new theory with strong connections with geometric combinatorics, semigroup theory and other areas of classical algebra. This theory has been the focus of my attention for the past few years and I have been working on it in close collaboration with Swapneel Mahajan. I will start by discussing a few basic notions pertaining to real hyperplane arrangements, focusing on the Tits product of faces. Then I will try to support the central claim by defining extensions of the notion of Hopf algebra, Lie algebra, and operads. I will mention extensions of a few selected results such as a theorem of Joyal, Klyachko and Stanley (relating the free Lie algebra to the partition lattice), the Cartier-Milnor-Moore theorem (relating Hopf and Lie algebras), and concepts such as Koszul duality. The conclusion is that much of this classical theory admits an extension relative to a real hyperplane arrangement.

Joint work with Swapneel Mahajan (Indian Institute of Technology Mumbay, India).

Plenary talk - July 26, 9:00 – 9:50

Elliptic fibrations and K3 surfaces

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Among algebraic surfaces, those that have an elliptic fibration, i.e., that are endowed with a proper morphism to a smooth curve whose fibers are, almost all, elliptic curves, play a special role: they can be regarded as an elliptic curve over the function field of the base curve and as a family of curves over the base curve. This two folded description makes such objects simultaneously intriguing and simpler to treat.

If one classifies algebraic surfaces by Kodaira dimension, one finds elliptic surfaces in all (Kodaira) dimensions but for $\kappa = 2$. But the only subclass that might admit more than one elliptic fibration with a section is that of K3 surfaces. It is therefore natural to search for a classification of elliptic fibrations on K3 surfaces.

I this talk I will introduce the definitions above and discuss the classification of elliptic fibrations on K3 surfaces, focusing, towards the end, in a special class given by the ones endowed with a non-symplectic involution.

Joint work with Alice Garbagnati (Univ. Milano).

Plenary talk - July 26, 10:30 – 11:20

Topological methods to solve equations over groups

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We present a large class of groups (no group known to be not in the class) that satisfy the Kervaire-Laudenbach Conjecture about solvability of non-singular equations over groups. We also show that certain singular equations with coefficients over groups in this class are always solvable. Our method is inspired by seminal work of Gerstenhaber-Rothaus, which was the key to prove the Kervaire-Laudenbach Conjecture for residually finite groups. Exploring the structure of the p-local homotopy type of the projective unitary group, we manage to show that many singular equations with coefficients in unitary groups can be solved in the unitary group.

Joint work with Anton Klyachko.

Plenary talk - July 26, 11:30 – 12:20

Tame topology and Complex Analytic Geometry

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In early 1980's logicians L. van den Dries, A. Pillay and C.Steinhorn introduced o-minimal structures that can be viewed as a Model Theoretic solution to Grothendieck's program of developing tame topology.

In this talk we start with a brief discussion of Grothendick's idea of tame topology. As examples we consider semi-algebraic and sub-analytic geometries. We will also demonstrate as the tameness of sub-analytic sets can be used in the context of Complex Analytic Geometry.

Joint work with Y.Peterzil.

Plenary talk - July 27, 10:00 – 10:50

On new applications of noncommutative rings

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We will mention some open questions and results in noncommutative ring theory related to other research areas; for example, related to resolutions of noncommutative singularities, superpotential algebras, Jacobi algebras, noncommutative projective algebraic geometry and group theory. Some new ring theoretic approaches for studying differential polynomial rings and tensor products $A \otimes A$ and $A \otimes A^{opp}$ will be mentioned within the context of coalgebras, Hopf algebras and Lie algebras. These methods are related to the Golod-Shafarevich theorem.

We will also look at a ring theoretic approach to the Yang-Baxter equation, which explores the connection between braces and nilpotent rings. Braces are a generalisation of Jacobson radical rings, and have been introduced by Rump as a tool for investigating non degenerate involutive set-theoretic solutions of the Yang-Baxter equation. We will present both old and new results from this area, together with a gentle introduction to the subject. No previous knowlege of braces, braided groups nor of the Yang-Baxter equation is assumed.

Plenary talk - July 27, 11:30 – 12:20

Arithmetic geometry of toric varieties

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There is a very rich theory linking the algebraic geometry of toric varieties with combinatorial properties. For instance, to a toric variety X provided with an ample divisor D we can associate a lattice polytope Δ. We can recover the variety and the divisor from the polytope and many properties of (X, D) can be read from this polytope. For instance the degree of D is given by $n!$ times the volume of the polytope ($n = \dim(X)$) and a basis of the global sections of $\mathcal{O}(D)$ is given by the integral points of the polytope. In a join project with P. Philippon and M. Sombra we have extended this toric dictionary to the Arakelov theory of toric varieties. Each toric variety has a canonical model over $\mathbb{Z}$. To a semipositive hermitian metric on $\mathcal{O}(D)$, invariant under the action of the compact torus, we associate a concave function ϑ on Δ. Called the roof function. The objective of this talk is to convince you that the roof function can be seen as an extended polytope that codifies most of the Arakelovian properties of X. For instance we can compute from it the height of X, the arithmetic volume of X, the essential and absolute minima and whether there is equidistribution of Galois orbits of small points.

Joint work with Martin Sombra (ICREA and Universitat de Barcelona), Patrice Philippon (CNRS and Institut de Mathématiques de Jussieu), Atsushi Moriwaki (University of Kyoto) and Juan Rivera-Letelier (University of Rochester).

Plenary talk - July 28, 9:00 – 9:50

Subgroups of the interval exchange transformation group.

Kate Juschenko
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I will discuss several classes of subgroups of interval exchange transformation group (IET). These subgroups come from topological full groups of corresponding rotations. Amenability and absence of free subgroups are the main questions we are going to discuss with relation to the subgroups of IET.

Plenary talk - July 28, 10:30 – 11:20

A_∞-algebras in representation theory and homological algebra

Estanislao Herscovich
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In this talk I will recall the basic theory of A_∞-algebras, which were introduced by J. Stasheff in 1963. Even though they appeared within the realm of algebraic topology, I will present several examples of their use in representation theory and in homological algebra. In particular, in the last part of the talk I will show that the torsion theory of A_∞-algebras naturally allows to compute the cup and cap products on Hochschild cohomology and homology of any nonnegatively graded connected algebra, respectively.

Plenary talk - July 28, 11:30 – 12:20

Cluster algebras and quantum affine algebras

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Quantum affine algebras have a rich theory of finite-dimensional representations, with important applications to integrable systems in statistical mechanics and quantum field theory. In 2008, it was discovered that the Grothendieck rings of certain monoidal categories of representations have the natural structure of a cluster algebra, such that all cluster monomials are classes of irreducible representations.

Since then the theory has developed a lot. It now covers all untwisted quantum affine algebras and larger classes of representations. One recent application is a geometric formula for the q-character of a product of Kirillov-Reshetikhin modules, in terms of Euler characteristics of certain new types of quiver varieties.

I will give a survey of the main results and conjectures, and if time allows, I will mention some recent extension of the theory to some infinite-dimensional representations of Borel subalgebras of quantum affine algebras.

Joint work with David Hernandez (Université Paris 7, France).

Plenary talk - July 29, 10:00 – 10:50

Sum-product estimates in finite fields

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The sum-product phenomenon, due to Erdös and Szemerédi, asserts, roughly speaking, that for any set A of integers either the sum set $A+A$ or the product set AA has the cardinality significantly larger than the cardinality of A. A finite field analogue of this problem was solved in 2003 by Bourgain, Katz and Tao. The sum-product estimate and its versions have found important applications in various areas of mathematics.

In this talk I will discuss sum-product estimates in finite fields and show some of their applications.

Plenary talk - July 29, 11:30 – 12:20

Tensors and their Eigenvectors

Bernd Sturmfels
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Eigenvectors of square matrices are central to linear algebra. Eigenvectors of tensors are a natural generalization. The spectral theory of tensors was pioneered by Lim and Qi a decade ago, and it has found numerous applications. We present an introduction to this theory, with focus on results on eigenconfigurations due to Abo, Cartwright, Robeva, Seigal and the author. We also discuss a count of singular vectors due to Friedland and Ottaviani.
