XXI CLA - Session S02
Commutative Algebra and Algebraic Geometry

S02 - July 25, 15:00 – 15:30

Ideals of fat points: subhomaloidal types and symbolic powers

Aron Simis
Universidade Federal de Pernambuco, Brazil
aron@dmat.ufpe.br

The subject theme relates to the intertwinning between plane Cremona maps and ideals of plane fat points, taking up both algebraic and geometric developments. The classical terminology "homaloidal types" refers to the virtual multiplicities of the base points of a Cremona map, while the ones on the title are closely related thereof and the associated ideal of fat points has interesting properties. The notion binding the two types together is that of the (second) symbolic power. The talk will give a glimpse of the homological facet involved as well as the relation to the classical Bordiga—White varieties.

Joint work with Zaqueu Ramos (Universidade Federal de Sergipe, Brazil).

S02 - July 25, 15:40 – 16:10

Tannaka duality for algebraic groups

Alvaro Rittatore
CMAT-Udelar, Uruguay
alvaro@cmat.edu.uy

The Chevalley's structure theorem states that any connected algebraic group over an algebraically closed field is the extension of an abelian variety by a connected affine algebraic group. In view of this result, the theory of algebraic groups has been developed along two directions: the study of linear (affine) algebraic groups and that of abelian varieties. The representation theory of affine algebraic groups plays an important role in their study: the (classical) Tannaka duality theorem guarantees that an affine algebraic group can be recovered from its category or representations.

In this talk we propose a representation theory for arbitrary algebraic groups, as follows: let G be an algebraic group G consider its Chevalley decomposition $1 \to G_{aff} \to G \to A \to 0$. A representation of G is a homogeneous vector bundle $E \to A$ together with regular action $\varphi : G \times E \to E$, linear on the fibres and such that the induced morphism $\widetilde{\varphi} : A \times A \to A$ is the product in A (recall that $A \times A$ is the Albanese variety of $G \times E$). We will define the category of representations of G, and prove that a generalisation of Tannaka duality theorem is valid in this context, therefore allowing us to recover an algebraic group from its category of representations.

This is an ongoing joint work, partially financed by CSIC-Udelar and ANII (Uruguay).

Joint work with Pedro Luis del Ángel (CiMat, México).

S02 - July 25, 16:20 – 16:50

A Surface Coming from an Arithmetic Question

Homero Gallegos-Ruiz
CONACyT - Universidad Autonoma de Zacatecas, Mexico
h.r.gallegos.ruiz@gmail.com

We study a surface in $\mathbb{P}^6$, which is a complete intersection of four quadrics coming from the rational distance problem: given a unit square on the plane, is there a point on the plane whose distances to the four points are all rational?

Joint work with Martha Bernal (CONACyT - Universidad Autonoma de Zacatecas, Mexico.).

S02 - July 25, 17:30 – 18:00

TBA

Louis Rowen
Bar-Ilan University, Israel
rowen@macs.biu.ac.il

S02 - July 25, 18:10 – 18:40

How to identify Milnor fibers of smoothings of quotient sigularities

Giancarlo Urzúa
Pontificia Universidad Católica de Chile, Chile
urzua@mat.uc.cl

This is part of my joint paper "Milnor fibers and symplectic fillings of quotient surface singularities" (arXiv:1507.06756) with Heesang Park, Dongsoo Shin and Jongil Park. I will explain how MMP is used to identify the Milnor fiber of a smoothing of a 2-dimensional quotient singularity. This is used to give a geometrical one-to-one correspondence between Milnor fibers and certain zero continued fractions, for the case of cyclic quotient singularities, which recovers the correspondence of Kollár-Shepherd-Barron, Christophersen-Stevens, and Lisca (connecting Milnor fibers with symplectic fillings). The MMP used is a small part of a bigger explicit MMP for families of surfaces described in my joint paper "Flipping surfaces" with Paul Hacking and Jenia Tevelev.

Joint work with Heesang Park (Konkuk University, South Korea), Dongsoo Shin (Chungnam National University, South Korea) and Jongil Park (Seoul National University, South Korea).

S02 - July 25, 18:50 – 19:20

Shapes of the simplest minimal free resolutions in $\mathbb{P}^1 \times \mathbb{P}^1$

Nicolás Botbol
Universidad de Buenos Aires - CONICET, Argentina
nbotbol@dm.uba.ar

The study of graduate syzygies for the standard case 3 homogeneous polynomials is now well known, but the general context (especially in the multihomogeneous case) is of greater interest but involves further difficulties and is very much unknown.

In 2007, Cox, Dickenstein and Schenck have analysed in depth, from both a geometric and an algebraic perspective, the minimal free resolution of an ideal given by 3 bihomogeneous polynomials of bidegree $d = (2, 1)$ in $R = k[X_1, X_2][X_3, X_4]$. In particular, an accurate description of the non-Koszul syzygies is obtained from an application of the Künneth formula and the Serre duality for $\mathbb{P}^1$; see the 1994 paper by Weyman and Zelevinsky on the subject.

In this talk, our goal is to give a detailed description of the (multi)graded minimal free resolution of an ideal I of R, generated by 3 bihomogeneous polynomials defined by $\mathbf{f} = (f_1, f_2, f_3)$ with bidegree (d_1, d_2), $d_i > 0$ and such that $V(I)$ is empty in $\mathbb{P}^1 \times \mathbb{P}^1$. We will precise the shape of the resolution in degree $d = (1, n)$, and explain how non-genericity (factorization) of the f_i's determine the resolution.

Joint work with Alicia Dickenstein (Universidad de Buenos Aires, Argentina) and Hal Schenck (University of Illinois, USA).

S02 - July 26, 15:00 – 15:30

Graded rings associated to valuations

Steven Dale Cutkosky
University of Missouri, USA
cutkoskys@missouri.edu

Suppose that K is a field with a valuation v and R is a local ring of K which is dominated by v. We discuss various graded rings associated to the valuation on R which provide fundamental information about birational properties of the ring. We consider especially differences between characteristic zero and positive characteristic and the question of finite generation and relative finite generation in a finite extension.

S02 - July 26, 15:40 – 16:10

The Birational Geometry of $\overline{M}_{0,6}$

Martha Bernal
CONACyT - Universidad Autonoma de Zacatecas, Mexico
m.m.bernal.guillen@gmail.com

We give relations among the Castravet's generators of the Cox ring of $\overline{M}_{0,6}$ and describe the maps represented by those relations.

S02 - July 26, 16:20 – 16:50

Bernstein–Sato polynomials for ideals in semigroup rings

Laura Felicia Matusevich
Texas A&M University, USA
laura@math.tamu.edu

We extend the Budur–Mustaţă–Saito definition of Bernstein–Sato polynomials for varieties, to the context of ideals in normal semigroup rings. In the special case of monomial ideals in normal semigroup rings, we also provide a correspondence between certain roots of the Bernstein–Sato polynomial and certain jumping coefficients of the corresponding multiplier ideals.

Joint work with Jen-Chieh Hsiao (National Cheng Kung University, Taiwan).

S02 - July 26, 17:30 – 18:00

Mod 2 cohomology rings of moduli stacks of Real vector bundles

Florent Schaffhauser
Universidad de los Andes, Colombia
florent@uniandes.edu.co

The rational cohomology ring of the moduli stack of holomorphic vector bundles of fixed rank and degree over a compact Riemann surface was studied by Atiyah and Bott using tools of differential geometry and algebraic topology: they found generators of that ring and computed its Poincaré series. In joint work with Chiu-Chu Melissa Liu, we study in a similar way the mod 2 cohomology ring of the moduli stack of Real vector bundles of fixed topological type over a compact Riemann surface with Real structure. The goal of the talk is to explain the principle of the computation, emphasizing the analogies and differences between the Real and complex cases, and discuss applications of the method. In particular, we provide explicit generators of mod 2 cohomology rings of moduli stacks of vector bundles over a real algebraic curve.

S02 - July 26, 18:10 – 18:40

The algebraic density property for affine toric varieties

Alvaro Liendo
Universidad de Talca, Chile
aliendo@inst-mat.utalca.cl

In this talk we generalize the algebraic density property to notnecessarily smooth affine varieties relative to some closed subvariety containing the singular locus. This property implies the remarkable approximation results for holomorphic automorphisms of the Andersén-Lempert theory. We show that an affine toric variety X satisfies this algebraic density property relative to a closed T-invariant subvariety Y if and only if the complement of Y in X is different ffrom T. For toric surfaces we are able to classify those which posses a strong version of the algebraic density property (relative to the singular locus). The main ingredient in this classification is our proof of an equivariant version of Brunella's famous classification of complete algebraic vector fields in the affine plane.

Joint work with Frank Kutzschebauch (Bern Universität, Switzerland) and Matthias Leuenberger (Bern Universität, Switzerland).

S02 - Poster

Rank two Vector bundles with first cohomology module generated by two elements and applications

Charles Aparecido Almeida
University of Campinas, Brazil
charles@ime.unicamp.br

We present a family of monads whose cohomology are μ-stable vector bundles of small rank on $\mathbb{P}^3$, whose first module of cohomology is generated by two elements, then study the geometrical properties of this family on the moduli space of stable vector bundles over $\mathbb{P}^3$. We use these results to show that the moduli space of stable rank two vector bundles with zero first Chern class and five second Chern class has exactly 3 irreducible components.

Joint work with Marcos Jardim (University of Campinas).

S02 - Poster

Intersections of Amoebas

Timo de Wolff
Texas A&M University, USA
dewolff@math.tamu.edu

Given an Laurent polynomial $f \in \mathbb{C}[z_1^{\pm 1}, \ldots, z_n^{\pm 1}]$ the amoeba $\mathcal{A}(f)$ (introduced by Gelfand, Kapranov, and Zelevinsky '94) is the image of its variety $\mathcal{V}(f) \subseteq (\mathbb{C}^*)^n = (\mathbb{C} \setminus \{0\})^n$ under the $\mathrm{Log}|\cdot|$-map

$$\mathrm{Log}|\cdot| : (\mathbb{C}^*)^n \to \mathbb{R}^n, \quad (z_1, \ldots, z_n) \mapsto (\log|z_1|, \ldots, \log|z_n|).$$

Amoebas have amazing structural properties; they are related to various mathematical subjects like complex analysis, the topology of real algebraic curves, nonnegativity of polynomials, dynamical systems, and particularly tropical geometry.

While amoebas of hypersurfaces have been studied intensively during the last years, the non-hypersurface case is not understood so far. Here, we investigate intersections of amoebas of n hypersurfaces in $(\mathbb{C}^*)^n$, which are canonical supersets of amoebas given by non-hypersurface varieties. As a main result we present an amoeba analog of the classical Bernstein Theorem from combinatorial algebraic geometry providing an upper bound for the number of connected components of such intersections.

We also show how the order map for hypersurface amoebas can be generalized in a natural way to intersections of amoebas. Particularly, analogous to the case of amoebas of hypersurfaces, the restriction of this generalized order map to a single connected component of the intersection is still 1-to-1.

For further information see http://arxiv.org/abs/1510.08416.

Joint work with Martina Juhnke-Kubitzke (Universität Osnabrück).

S02 - Poster

Rational Harnack Curves on Toric Surfaces

Jorge Alberto Olarte Parra
Universidad de los Andes, Colombia
ja.olarte1299@uniandes.edu.co

Harnack curves are a family of real algebraic curves who are distinguished because their topology is well understood, meaning that Hilbert's 16th problem is solved for these curves. Let f be a 2 variable real

polynomial whose Newton polygon is Δ and let C be the curve defined as the zeros of f inside the toric variety X_Δ. The original definition of Harnack curves by Mikhalkin states that the real part of C, $\mathbb{R}C \subseteq \mathbb{R}X_\Delta$, is a Harnack curve if and only if the following conditions are satisfied:

1. The number of connected components of $\mathbb{R}C$ is maximal, that is $g+1$, where g is the arithmethic genus of C.

2. Only one component O intersects the axes of $\mathbb{R}X_\Delta$.

3. Let $l_1, \ldots, l_n$ be the axes of X_Δ ordered in a way such that it agrees with the cyclical order of their corresponding sides of Δ and let $d_1, \ldots, d_n$ be the integer lengths of the corresponding sides. Then O can be divided into disjoint arcs $\alpha_1, \ldots \alpha_n$ such that $\alpha_i \cap l_i = d_i$ and $\alpha_i \cap l_j = 0$ when $j \neq i$.

These curves have several different characterizations, for example, its amoeba (the image of C under the map $(z, w) \mapsto (\log|z|, \log|w|)$) is of maximal area. These curves have applications to physics through dimer theory. In this poster we focus on rational Harnack curves, which are Harnack curves of genus 0 and we show how these curves can be explicitly parametrized using the homogeneous coordinates of X_Δ.

Joint work with Mauricio Velasco (Universidad de los Andes, Colombia).

S02 - Poster

Orthogonal instanton bundles of higher rank on $\mathbb{P}^3$

Aline Vilela Andrade
University of Campinas, Brazil
aline.andrade@ime.unicamp.br

In this work we provide a bijection between equivalence classes of orthogonal instanton bundles over $\mathbb{P}^3$ and symmetric forms. Using such correspondence, we prove the non-existence of orthogonal instanton bundles on $\mathbb{P}^3$, with second Chern class equals to one or two, and we also provide examples of orthogonal instanton bundles of second Chern classes three and four on $\mathbb{P}^3$.

Joint work with Simone Marchesi (University of Campinas, Brazil).
