XXI CLA - Workshop S03
Hopf Algebras

S03 - July 25, 15:00 – 15:50

Nichols algebras of finite Gelfand-Kirillov dimension over abelian groups

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I will give an overview of the classification of Nichols algebras of finite GK-dimension over abelian groups, including numerous new examples.

Joint work with Iván Angiono (Universidad Nacional de Córdoba) and István Heckenberger (Universität Marburg).

S03 - July 25, 16:00 – 16:25

The classification of Nichols algebras

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Nichols algebras appear in several branches of mathematics going from Hopf algebras and quantum groups, to Schubert calculus and conformal field theories. In this talk we review the main problems related to Nichols algebras and we discuss some classification theorems. The talk is mainly based on joint works with I. Heckenberger.

Joint work with István Heckenberger (Philipps-Universität, Marburg, Germany).

S03 - July 25, 16:30 – 16:55

On rigidity of Nichols algebras

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Nichols algebras have become one of the main objects in the theory of Hopf algebras. In particular, Nichols algebras of diagonal type play a central role in the classification program for pointed Hopf algebras, which are obtained by the two-step process of bosonization and graded deformation (so called "lifting") from these Nichols algebras.

It turns out that if the first step of this process is omitted then there may well be no graded deformations. More precisely, certain wide classes of graded braided bialgebras, including finite-dimensional Nichols algebras of diagonal type, positive parts of quantum groups, and finite-dimensional symmetric algebras

of braided vector spaces whose braiding comes from the (co)action of a finite-dimensional (co)triangular Hopf algebra, do not admit nontrivial graded deformations.

Joint work with Iván Angiono (Universidad Nacional de Córdoba, Argentina) and Mitja Mastnak (Saint Mary's University, Canada).

S03 - July 25, 17:30 – 17:55

Partial actions of Hopf algebras: how it all started

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This talk is a survey about the development of the study of partial Hopf actions since its beginning.

S03 - July 25, 18:00 – 18:25

Tropical Hopf algebras and weak Hopf algebras

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Recently tropical versions of Lie algebras have been developed. In this talk we present the foundation of the tropical Hopf theory (over the max-plus algebra), and more generally Hopf algebras over semirings.

Joint work with Sara Westreich (Bar-Ilan University, Israel) and M. Cohen (Ben-Gurion University, Israel).

S03 - July 25, 18:30 – 18:55

A d.g. Hopf algebra associated to a set theoretical solution of the Yang-Baxter equation and cohomology

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For a set theoretical solution of the Yang-Baxter equation (X, r), we define a d.g. Hopf algebra $B = B(X, r)$ containing the group algebra $k[G]$, where $G = \langle x \in X : xy = zt \text{ if } r(x, y) = (z, t)\rangle$, in such a way that $k \otimes_G B \otimes_G k$ and $\mathrm{Hom}_{G-G}(B, k)$ are respectively the homology and cohomology complexes computing quandle/rack homology and cohomology, as defined by knot theorists (Carter, Saito, Jelsovskyb, ElHamadi) and other generalizations of cohomology (e.g. twisted rack cohomology, or Yang-Baxter cohomology). This algebraic structure allow us to show the existence of an associative product in Yang-Baxter cohomology, and a comparison map with Hochschild (co)homology of k[G], that factors trough the Nichols algebra associated to $(X, -r)$.

Joint work with Juliana Garcia Galofre (Universidad de Buenos Aires, Argentina)..

S03 - July 26, 15:00 – 15:50

On Hopf orders and Kaplansky's sixth conjecture

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A theorem of Frobenius states that the degree of any complex irreducible representation of a finite group G divides the order of G. This is proved using the following specific property of the group algebra $\mathbb{C}G$: it is defined over $\mathbb{Z}$ or, in other words, the group ring $\mathbb{Z}G$ is a Hopf order of $\mathbb{C}G$.

Kaplansky's sixth conjecture predicts that Frobenius Theorem holds for complex semisimple Hopf algebras. There are several partial results in the affirmative. Compared to the case of groups, the main difficulty to prove this conjecture (if true) is that it is not guaranteed that a complex semisimple Hopf algebra H is defined over $\mathbb{Z}$ or, more generally, over a number ring. If it would be so, Larson proved that H satisfies Kaplansky's sixth conjecture. The question whether every complex semisimple Hopf algebra can be defined over a number ring has always been behind this conjecture.

In this talk we will answer this question in the negative. The family of examples that we will handle, constructed by Galindo and Natale, are Drinfeld twists of certain group algebras. The key fact is that the twist contains a scalar fraction, which makes impossible to define such Hopf algebras over a number ring.

The results that will be presented are part of a joint work with Ehud Meir (University of Hamburg) published in Trans. Amer. Math. Soc. and available at arXiv.org.

S03 - July 26, 16:00 – 16:25

Universal quantum groups associated to a pair of preregular forms

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I will discuss recent joint work with Alexandru Chirvasitu and Xingting Wang (arXiv:1605.06428), where we define the universal quantum group $\mathcal{H}$ that preserves a pair of Hopf comodule maps whose underlying vector space maps are preregular forms defined on dual vector spaces. This generalizes the construction of Bichon and Dubois-Violette (2013), where the target of these comodule maps are the ground field. We also recover the quantum groups introduced by Dubois-Violette and Launer (1990), by Takeuchi (1990), by Artin, Schelter, and Tate (1991), and by Mrozinski (2014), via our construction. As a consequence, we obtain an explicit presentation of a universal quantum group that coacts simultaneously on a pair of N-Koszul Artin-Schelter regular algebras with arbitrary quantum determinant.

Joint work with Alexandru Chirvasitu (University of Washington, Seattle, United States) and Xingting Wang (Temple University, United States).

S03 - July 26, 16:30 – 16:55

Noncommutative discriminants via Poisson primes

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Discriminants play a key role in various settings in algebraic number theory, algebraic geometry, combinatorics, and noncommutative algebra. In the last case, they have been computed for very few algebras. We will present a general method for computing discriminants of noncommutative algebras which is applicable to algebras obtained by specialization from families, such as quantum algebras at roots of unity. It builds a connection with Poisson geometry and expresses the discriminants as products of Poisson primes.

Joint work with Bach Nguyen and Kurt Trampel (Louisiana State University).

S03 - July 26, 17:30 – 17:55

Non-associative exponentials and the Baker–Campbell–Hausdorff formula

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An exponential map is a power series in one variable that sends the set of primitive elements in a complete Hopf algebra to the set of its group-like elements, and whose linear term has coefficient 1. While the exponential map is unique in the associative setting, in the non-associative case there are infinitely many exponential maps.

In this talk I will describe the set of all non-associative exponential maps as a torsor for a certain residually nilpotent group. I will also talk about the problem of constructing the non-associative version of the Dynkin form of the Baker-Campbell-Hausdorff formula; that is, expressing $\log(\exp(x)\exp(y))$, where x and y are non-associative variables, in terms of the Shestakov-Umirbaev primitive operations.

Joint work with J.M. Pérez Izquierdo (Universidad de la Rioja, España) and I.P. Shestakov (Universidade de São Paulo, Brasil).

S03 - July 26, 18:00 – 18:25

Pointed and copointed Hopf algebras over dihedral groups

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Let k be an algebraically closed field of characteristic 0 and let D_m be the dihedral group of order $2m$ with $m = 4t$; $t \geq 3$. This talk will be based on joint work with Fernando Fantino and Mitja Mastnak [FG], [GM] and [FGM] where we classify all finite-dimensional pointed and copointed Hopf algebras whose group of group-likes is D_m by means of the lifting method and 2-cocycle deformations. As a byproduct we obtain new examples of finite-dimensional pointed and copointed Hopf algebras.

Among many useful tools for constructing new Hopf algebras is the use of multiplicative 2-cocycles for deforming the multiplication of a given Hopf algebra (and the dual notion of deforming its coproduct by using a twist). With this in mind, it is interesting to ask whether some non-isomorphic Hopf algebras might be cocycle deformations of each other. It has been proven by different methods that all known families of finite-dimensional pointed and copointed Hopf algebras over abelian and non-abelian groups can be constructed by deforming the multiplication of bosonizations of Nichols algebras.

It turns out that it is also the case for all pointed and copointed Hopf algebras over D_m. We show this result by giving explicitly the family of 2-cocycles that give the deformation. Besides introducing these

families of Hopf algebras, I will describe how to produce such cocycles and give the appropriate setting where the construction applies.

References

[FG] F. Fantino and G. A. García, On pointed Hopf algebras over dihedral groups. Pacific J. of Math. Vol. 252 (2011), no. 1, 69–91.

[FGM] F. Fantino, G. A. García and M. Mastnak, On copointed Hopf algebras over dihedral groups. In preparation.

[GM] G. A. García and M. Mastnak, Deformation by cocycles of pointed Hopf algebras over non-abelian groups, Math. Res. Lett. 22 (2015), 59–92.

Joint work with Fernando Fantino (Universidad Nacional de Córdoba, Argentina) and Mitja Mastnak (St. John's University, Canada).

S03 - July 26, 18:30 – 18:55

The five-term exact sequence for Kac cohomology

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The set of equivalence classes of abelian extensions of Hopf algebras associated to a matched pair of finite groups turns out to be a group. This group was described by Kac in the 60's as the second cohomology group of a double complex, whose total cohomology is known as the Kac cohomology. Masuoka generalized this result and used it to construct and classify semisimple Hopf algebra extensions. Since Kac cohomology is defined as the total cohomology of a double complex, there is an associated spectral sequence. We compute the second page of this spectral sequence and the five-term exact sequence associated. Through some examples we show how this new exact sequence is very useful to compute the group of abelian extensions.

Joint work with Yiby Morales (Universidad de los Andes).

S03 - Poster

Representation of weak Hopf algebra arising from fusion double groupoids

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Let $\Bbbk$ be a field and $\operatorname{Rep}\Bbbk\mathcal{T}$ be the category of finite dimensional representations of the weak Hopf algebra $\Bbbk\mathcal{T}$ associated to a finite fusion double groupoid $\mathcal{T}$. In this work we study proprieties of $\operatorname{Rep}\Bbbk\mathcal{T}$. More precisely, we investigate when this category is group-theoretical.

Joint work with Nicolás Andruskiewitsch (Universidad Nacional de Córdoba, Argentina) and Daiana Flôres (Universidade Federal de Santa Maria).

S03 - Poster

Quantum Subgroups of Simple Twisted Quantum Groups at Roots of One

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Let G be a connected, simply connected simple complex algebraic group and let ϵ be a primitive ℓth root of unity with ℓ odd and coprime with 3 if G is of type G_2. We determine all Hopf algebra quotients of the twisted multiparameter quantum function algebra $\mathcal{O}_\epsilon^\varphi(G)$ introduced by Costantini and Varagnolo. This extends the results of Andruskiewitsch and the first author, where the untwisted case is treated.

Joint work with Gastón Andrés García (Universidad Nacional de La Plata).

S03 - Poster

Examples of finite-dimensional Hopf algebras with the dual Chevalley property.

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We present examples of Hopf algebras with the dual Chevalley property, that is, a Hopf algebra whose coradical is a Hopf subalgebra. For this, we determine all semisimple Hopf algebras Morita-equivalent to a group algebra over a finite group, for a list of groups supporting a non-trivial finite-dimensional Nichols algebra.

Joint work with A. Andruskiewitsch (Universidad Nacional de Córdoba, Argentina) and C. Galindo (Universidad de los Andes, Colombia).

S03 - Poster

Hopf automorphisms and twisted extensions

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In this poster, I will describe some applications of a Hopf algebra constructed from a group acting on another Hopf algebra A as Hopf automorphisms, namely Molnar's smash coproduct Hopf algebra. I will also describe connections between the exponent and Frobenius–Schur indicators of a smash coproduct and the twisted exponents and twisted Frobenius-Schur indicators of the original Hopf algebra A.

Joint work with Susan Montgomery (University of Southern California, United States) and Sarah Witherspoon (Texas A&M University, United States).
