XXI CLA - Session S05
Rings and Algebras

S05 - July 28, 15:00 – 15:45

Octonions in low characteristics

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Some special features of Cayley algebras, and their Lie algebras of derivations, over fields of low characteristics will be presented. As an example, over fields of characteristic two, the isomorphism class of the Lie algebra of derivations of a Cayley algebra does not depend on the Cayley algebra itself.

S05 - July 28, 15:45 – 16:10

Polynomial identities, codimensions and a conjecture of Regev

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Let A be an algebra over a field F of characteristic zero and $Id(A)$ its T-ideal of identities. The space of multilinear polynomials in n fixed variables modulo $Id(A)$ is a representation of the symmetric group S_n and its degree is called the nth codimension of A. As soon as A is associative and satisfies a non-trivial identity, its sequence of codimensions is exponentially bounded and, following a conjecture of Amitsur regarding its exponential growth, Regev made a conjecture about the precise asymptotics of such sequence. I will talk about the results around this conjecture also in the case of non associative algebras.

S05 - July 28, 16:10 – 16:35

Essencial idempotents in group algebras and coding theory

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We introduce the concept of essencial idempotents in group algebras, a notion inspired in coding theory. We shall give some criteria to identify which primitive idempotents are essential, and discuss some applications. Among these, we show that every minimal non-cyclic abelian code is a repetition code, and that every minimal abelian code is combinatorially equivalent to a cyclic code of the same lenth. Also, we shall give an example showing that a non minimal abelian code of lenth p^2 with p a prime integer, can be more convenient than any cyclic code of that length.

S05 - July 28, 16:35 – 17:00

Free groups in a normal subgroup of the field of fractions of a skew polynomial ring

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Let $k(t)$ be the field of rational functions over the field k, let σ be a k-automorphism of $K = k(t)$, let $D = K(X;\sigma)$ be the ring of fractions of the skew polynomial ring $K[X;\sigma]$, and let $D^\bullet$ be the multiplicative group of D. We show that if N is a non central normal subgroup of $D^\bullet$, then N contains a free subgroup.

S05 - July 28, 17:30 – 17:50

Non-commutative algebraic geometry of semi-graded rings

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In this short talk we introduce the semi-graded rings, which generalize graded rings and skew PBW extensions. For this new type of non-commutative rings we will study some basic problems of non-commutative algebraic geometry. In particular, we will discuss the Serre-Artin-Zhang-Verevkin theorem about non-commutative schemes.

Joint work with Edward Latorre (Universidad Nacional de Colombia, Bogotá, Colombia).

S05 - July 28, 17:50 – 18:10

On finite generation and presentation of algebras

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The talk will focus on finite generation and presentation of associative and Lie algebras with idempotent conditions.

Joint work with Hamed Alsulami.

S05 - July 28, 18:30 – 18:50

On Matrix Rings with The SA Property

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In this paper, matrix rings with the SIP and the ads (briefly, SA) are studied. A ring R has the right summand intersection property (SIP) if the intersection of two direct summands of R is also a direct summand. A right R-module M has the absolute direct summand property (ads) if for every decomposition

$M = A \oplus B$ of M and every complement C of A in M, we have $M = A \oplus C$. Let R be any ring with identity, e an idempotent in R such that $R = ReR$ and S the subring eRe and $R = Mat_n(S)$. It is shown that R_R has the SA if and only if S^n_S has the SA. It is also shown with an example that the SA is not the Morita invariant property.

Keywords: Ads property, summand intersection property, trivial extension.

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S05 - July 29, 15:00 – 15:45

Finitely presented Lie and Jordan algebras

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We will consider important examples of Lie and Jordan algebras and address the question when they can be presented by finitely many defining relations.

S05 - July 29, 15:45 – 16:10

Graded algebras and polynomial identities

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Connections (or "bridges") between PI theory (polynomial identities) and group gradings on associative algebras are quite well known for more than 30 years. For instance, Kemer applied the theory of "super algebras" in order to solve the famous Specht problem for nonaffine PI algebras. Our interest is in the opposite direction. We apply PI theory in order to solve a conjecture of Bahturin and Regev on "regular G-gradings" on associative algebras where G is a finite abelian group. Moreover, we show how to extend it to nonabelian groups. As a second application, we present a Jordan's like theorem on G-gradings on associative algebras.

Joint work with Ofir David (Technion, Israel).

S05 - July 29, 16:10 – 16:35

Color involutions of primitive graded rings.

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Kaplansky's Theorem [2] characterizes involutions of primitive rings with a nonzero socle in terms of hermitian and alternate forms. In 1997 M.L.Racine [3] constructed similar structure theory for primitive associative superalgebras. And Yu.A. Bakhturin, M. Bresar, M. Kochetov [1] obtained similar results for graded rings with graded involutions.

We present analogous characterizations of primitive graded rings in terms of twisted pairing. This implies the extension of Kaplansky's Theorem for primitive graded rings with a color involution in case of a grading by a cyclic group of a prime order. We also obtain some corollaries on color involutions of finite dimensional simple graded algebras. In particular, these results generalise the corresponding theorems of [2].

The work is partially supported by CNPq, CAPES.

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Joint work with Keidna Cristiane Oliveira Souza (University of Brasilia, Brazil).

S05 - July 29, 16:35 – 17:00

Lie algebras of slow growth

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We discuss rather old and recent constructions of Lie algebras and superalgebras of slow growth. In particular, we obtain examples of finitely generated (self-similar) (restricted) Lie (super)algebras of slow polynomial growth with a nil p-mapping.

By their properties, these restricted Lie (super)algebras resemble Grigorchuk and Gupta-Sidki groups. We discuss different properties of these algebras and their associative hulls.

S05 - July 29, 17:30 – 17:55

On D algebras.

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Consider an algebraic function z of n variables $x_1, x_2, \ldots, x_n$. Denote by $D(z)$ a subalgebra of the field $\mathbb{C}(x_1, x_2, \ldots, x_n)[z]$ which is generated by $x_1, x_2, \ldots, x_n$; z and all partial derivatives of z. I am interested in properties of algebras $D(z)$.

In my talk I will discuss the following conjectural dichotomy:

If $z \in \mathbb{C}[x_1, \ldots, x_n]$ then (obviously) $D(z) = \mathbb{C}[x_1, \ldots, x_n]$, but if $z \notin \mathbb{C}[x_1, \ldots, x_n]$ then $D(z)$ cannot be embedded into a polynomial ring.

S05 - July 29, 18:00 – 18:25

Partial actions and subshifts

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An arbitrary (one-sided) subshift X over a finite alphabet Λ with n letters can be naturally endowed with a partial action θ of the free group $\mathbb{F}_n$ with n free generators $g_\lambda, (\lambda \in \Lambda)$, such that g_λ maps x to λx, where x is an element in X for which $\lambda x \in X$. Naturally g_λ^{-1} removes λ from λx. We call θ the standard partial action, and it is a starting point to constract a C^*-algebra $\mathcal{O}_X^*$ associated with X, as well as an abstract algbera $\mathcal{O}_X^K$ over an arbitrary field K of characteristic 0. Both $\mathcal{O}_X^K$ and $\mathcal{O}_X^*$ are defined in a fairly similar way: using the standard partial action we construct a partial representation u of $\mathbb{F}_n$ into an appropriate algebra (which depends on whether the case is abstract or C^*) and then define $\mathcal{O}_X^K$ (or $\mathcal{O}_X^*$) as the subalgebra (respectively, a C^*-subalgebra) generated by $u(\mathbb{F}_n)$. Then using a general procedure (see [4,Proposition 10.1] we obtain a partial action τ of $\mathbb{F}_n$ on a commutative subalgebra $\mathcal{A}$ and prove that $\mathcal{O}_X^K$ is isomorphic to the crossed product $\mathcal{A} \rtimes_\tau \mathbb{F}_n$. In the C^* case (see [3,Theorem 9.5]), due to an amenability property, $\mathcal{O}_X^*$ is isomorphic to both the full and the reduced crossed product: $\mathcal{O}_X^* \cong \mathcal{D} \rtimes_\tau \mathbb{F}_n \cong \mathcal{D} \rtimes_\tau^{\text{red}} \mathbb{F}_n$, where $\mathcal{D}$ is a commutative C^*-algebra defined in a similar way as $\mathcal{A}$. This gives a possibility to study algebras related to subshifts using crossed products by partial actions. It turns out that $\mathcal{O}_X^*$ is isomorphic to the C^*-algebra defined by T. M. Carlsen in [1] in a somewhat different way (see [3,Theorem 10.2]). In particular, if X is a Markov subshift, then $\mathcal{O}_X^*$ is isomorphic to the Cuntz-Krieger algebra defined in [2]. The C^* version is elaborated in the preprint [3], in which, amongst several related results, a criterion is given for simplicity of $\mathcal{O}_X^*$ (see [3,Theorem 14.5]).

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Joint work with Ruy Exel (Universidade Federal de Santa Catarina, Brazil).

S05 - July 29, 18:30 – 18:55

Identities of finitely generated alternative and Malcev algebras

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We prove that for every natural number n there exists a natural number $f(n)$ such that every multilinear skew-symmetric polynomial on $f(n)$ variables which vanishes in the free associative algebra vanishes as well in any n-generated alternative algebra over a field of characteristic 0. Similarly, for any n there exists $g(n)$ such that every multilinear skew-symmetric polynomial on $g(n)$ variables vanishes in any n-generated Malcev algebra over a field of characteristic 0. Before a similar result was known only for a series of skew-symmetric polynomials of special type on $2m+1$ variables constructed by the author, where $m > \frac{C_n^1+C_n^2+C_n^3}{2}$.

S05 - Poster

Abelian Group Codes

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Let F be a finite field and n, a non negative integer. A linear code C of length n is a subspace of F^n. A (left) group code of length n is a linear code which is the image of a (left) ideal of a group algebra via an isomorphism $FG \to F^n$ for any G, a finite group with $|G| = n$. In this case C, is a (left) G-code. In [1], Bernal, del Río and Simón obtain a criterion to decide when a linear code is a group code in terms of the group of permutation automorphisms of C, $PAut(C)$. Sabin and Lomonaco, in [4], have proved that if C a G-code with G a semidirect product of cyclic groups, then C is an abelian group code. As an application of criterion and extending the result of Sabin and Lomonaco, in [1], they provide a family of groups for which every two-sided group code is an abelian group code. Pillado, González, Martínez, Markov e Nechaev describe some classes of groups and fields for which all group codes are abelian in [2]. Motivated by [3], they have shown that there exist a non-Abelian G-code over F. In order to extend the result on groups with abelian decompotition, we explore some conditions to determine a group G which can written as a product of abelian subgroups, such that the G-codes with $G \in \mathcal{G}$ will be abelian group code.

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Joint work with Thierry Petit Lobão (Universidade Federal da Bahia, Brasil).

S05 - Poster

Semiclean Rings

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A ring R with unity is said to be clean if every element in the ring can be written as the sum of a unit and an idempotent of the ring. These rings were introduced by Nicholson, [**?**], in his study of lifting idempotents and exchange rings. The division rings, boolean rings and local rings are examples of clean rings.

In the article [**?**], a new class of rings is defined; semiclean rings . A ring R with unity is called semiclean if, every $x \in R$, $x = u + a$ with $u \in \mathcal{U}(R)$ where a is periodic element, i.e., $a^k = a^l$ with $k, z \in \mathbb{Z}$ and $k \neq z$. Therefore, every semiclean ring is a clean ring, because the idempotents elements of ring are periodics. Nicholson e Han, [**?**], demonstrated that group ring $Z_{(7)}C_3$ is not a clean ring. Yuanqing Ye showed, in the article [**?**], that the group ring $Z_{(p)}C_3$ is an semiclean ring. This result assures that the two classes, clean and semiclean, are different.

Motivated by the article [**?**], we intend to investigate if the Yuanging Ye's demonstration can be generalized, as in the cases $Z_{(11)}C_5$ and $Z_{(p)}C_5$, in search of a possible answer about the ring $Z_{(p)}C_q$ with p and q relatively primers.

Joint work with Elen Deise Assis Barbosa(Universidade Federal da Bahia, Brasil).

S05 - Poster

The Normalizer Property and its Relation with Extensions of Groups

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The determination of the normalizer of the basis group in the group of units of the associated group ring is a question that naturally imposes by itself. In integral group rings, in particular, it has been observed that, for important classes of finite groups, this normalizer is minimal, in other words, $\mathcal{N}_{\mathcal{U}}(G) = G \cdot Z$. When this occurs, we say that the group in question and its integral group ring satisfy the normalizer property. This property, also known as (Nor), has recently gained great importance when Mazur, in [Ma95], noticed an interesting relation with the famous problem of isomorphism in integral group rings also known as (Iso). Exploring this connection, Hertweck in [He01] found an example of a finite group that does not satisfy (Nor), and indirectly, by the relation mentioned above, obtained a counterexample to (Iso). Given that the counter example of Hertweck to (Nor) consists of an extension given by a semidirect product, but [LPS99] proves that extensions given by direct products are solutions (Nor), it is important to investigate which other other extensions of finite groups answer the property. Recently, Petit Lobão e Sehgal in [PeS03] demonstrated the validity of (Nor) for the class of complete monomial groups; in other words, a wreath extension of a finite nilpotent group with the symmetric group on m letters. Zhengxing Li e Jinke Hai in a series of articles, among which we have [HL12], [HL12b], HL11], also obtained interesting solutions of this property. The purpose of this work is to verify the relation between (Nor) and extensions of groups, where such component groups are solutions (Nor), in order to obtain necessary and sufficient conditions to find positive solutions to the property in question.

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Joint work with Thierry Petit Lobão (Universidade Federal da Bahia).

S05 - Poster

Involution inverting gradings on matrix algebras

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Let F be an algebraically closed field of characteristic zero, and G be a finite abelian group. If $M_n(F)$ is an algebra with involution $*$, we describe G-gradings $M_n(F) = A = \oplus_{g\in G} A_g$ on A, satisfying $(A_g)^* \subseteq A_{g^{-1}}$, for all $g \in G$.

Joint work with Luís Felipe Gonçalves Fonseca (Universidade Federal de Viçosa).

S05 - Poster

Anticommutativity of Symmetric and Skew-symmetric Elements under Generalized Oriented Involutions

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Given an involution $*$ in a group ring RG, we can define the sets $(RG)_* = \{\alpha \in RG : \alpha^* = \alpha\}$ and $(RG)_*^- = \{\alpha \in RG : \alpha^* = -\alpha\}$, called the set of symmetric and skew-symmetric elements, respectively. Under certain conditions in R, G, or the involution in RG, many authors proved that some identities satisfied in these sets could be lifted to the entire group ring, and, in some cases, given the impossibility of such lifting, they describe the basic structures of the group ring RG.

Generalizing the results found in [GP13a, GP13b, GP14], using a group homomorphism $\sigma : G \to \mathcal{U}(R)$, we will define and explore the involution $\sigma* : RG \to RG$, called generalized oriented involution, exposing the group structures, as well as the ring conditions, such that $(RG)_{\sigma*}$ or $(RG)^{-}_{\sigma*}$ be anticommutative.

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Joint work with Thierry Petit Lobao (Universidade Federal da Bahia, Brazil).

S05 - Poster

$*$ - Clean Group Algebras

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An element of a(n associative) ring (with 1) is clean if it is the sum of a unit and an idempotent. A ring is clean if every element in it is clean. The property of cleanness was formulated by Nicholson [4] in the course of his study of exchange rings. From then on, several related concepts were proposed: uniquely clean rings, strongly clean rings, weakly clean rings, $*$ - clean rings, r - clean rings, nil - clean rings, to cite a few. In the realm of group rings, these properties have been studied from 2001 [2] on with the aim of characterizing the rings R and groups G such that the group ring RG is clean.

In 2010 Vas proposed the definition of a $*$ - clean ring ("star"- clean) [5]: a $*$ - ring (i.e., rings with an involution) in which every element may be written as a sum of a unit and a projection. Clearly, every $*$-clean ring is a *star* - ring and is a clean ring. In [5], Vas asked: when is a $*$ - ring clean, but not $*$-clean? Every group G having an element g $\neq 1$, with $|\langle g\rangle| \neq 2$, is endowed with the classical involution $g \mapsto g^{-1}$. Because of that, group rings RG are almost always $*$ - rings: if R is a commutative rings, for instance, an involution in RG is obtained from the R - linear extension of the classical involution in G (and is also called the classical involution in RG). The $*$-cleanness of group rings was first approached in 2011 [3]. Even though some instances of group rings are answers to Vas's question [1], very little is still known about conditions under which a group ring with the classical involution is $*$-clean (not even the case of the group ring RG, where R is a commutative ring and G is a cyclic group, is fully stablished!).

In this talk, I present some recent results [1]. Let R be a commutative local ring. I will present RS_3 as an answer to Vas 's question, and I will provide necessary and sufficient conditions for the group ring RQ_8 to ber $*$ - clean, where Q_8 is the quaternion group of 8 elements.

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S05 - Poster

Irreducible components of varieties of Jordan algebras

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In 1968, F. Flanigan proved that every irreducible component of a variety of structure constants must carry an open subset of nonsingular points which is either the orbit of a single rigid algebra or an infinite union of orbits of algebras which differ only in their radicals.

In the context of the variety Jor_n of Jordan algebras, it is known that, up to dimension four, every component is dominated by a rigid algebra. In this work, we show that the second alternative of Flanigan's theorem does in fact occur by exhibiting a component of $JorN_5$ which consists of the Zariski closure of an infinite union of orbits of five-dimensional nilpotent Jordan algebras, none of them being rigid.

Joint work with Iryna Kashuba (Universidade de São Paulo, Brazil).

S05 - Poster

Construction of Rota^m-Algebras and Ballot^m-Algebras from Associative Algebras with a Rota-Baxter morphism and a Rota-Baxter Operator of Weights Three and Two

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We give a generalization of Rota-Baxter Operators and introduce the notion of a Ballot^m-algebra. Free Rota-Baxter algebras on a set can be constructed from a subset of planar rooted forests with decorations on the angles. We give similar constructions for obtaining an associative algebra in terms of planar binary trees with a modified Rota-Baxter Operator, and so we construct a Ballot^m-algebra.

We introduce the concepts of a Rota-Baxter Morphism, Dyck^m-algebra and Rota^m-algebra. An element u is said to be idempotent with respect to product $\cdot$ in the algebra if: $u \cdot u = u$, and it is a left identity if $x \cdot u = x$ for all element x in the algebra. Associative algebras with a left identity that simultaneously is a element idempotent, permit us to present examples of a Rota-Baxter Morphism and so we can construct a Rota^m-algebra.

We stress that the construction of Ballot^m-algebras and Rota^m-algebras from associative algebras with a generalitation of Rota-Baxter Operators are some of the main results of this work.

S05 - Poster

Clean rings and clean group rings

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A ring is said to be clean if each element in the ring can be written as the sum of a unit and an idempotent of the ring. The notion of a clean ring was introduced in 1977 by Nicholson in his study of lifting idempotents and exchange rings, and these rings have since been studied by many different authors.

In this poster, we present some properties and examples of clean rings, and then we classify the rings that consist entirely of units, idempotents, and quasiregular elements and we also consider the problems of classifying the groups G whose group rings RG are clean for any clean ring R.

Joint work with Rodrigo Lucas Rodrigues (Universidade Federal do Ceará).

S05 - Poster

On the max-plus algebra of non-negative exponent matrices

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An integer $n \times n$-matrix $A = (\alpha_{pq})$ is called exponent if all its diagonal entries are equal to zero and for all possible i, j and k the inequality $\alpha_{ij} + \alpha_{jk} \geqslant \alpha_{ik}$ holds. The study of exponent matrices is important because of their crucial role in the theory of tiled orders.

We show that the set $\mathcal{T}$ of minimal non-negative exponent $n \times n$-matrices can be described as follows. The matrix $T = (t_{ij}) \in \mathcal{E}_n$ belongs to $\mathcal{T}$ if and only if $t_{ij} \in \{0, 1\}$ for all i, j and there exists a proper subset $\mathcal{I}$ of $\{1, \ldots, n\}$ such that $t_{ij} = 1$ is equivalent to $i \in \mathcal{I}$ and $j \notin \mathcal{I}$.

Let $\oplus$ be the element-wise maximum of matrices and let $\otimes$ be a sum of matrices. Clearly, $A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C)$ for all A, B, $C \in \mathcal{E}_n$, whence $\mathcal{E}_n$ can be considered as an algebra $(\mathcal{E}_n, \oplus, \otimes)$, with respect to operations $\oplus$ and $\otimes$.

We prove the following result.

Theorem. *For any non-zero $A \in \mathcal{E}_n$ there exist a decomposition*

$$A = B_1 \otimes \ldots \otimes B_l \oplus \ldots \oplus C_1 \otimes \ldots \otimes C_m,$$

where all matrices $B_1, \ldots, C_m$ belong to $\mathcal{T}$ and as usual $\otimes$ performed prior to $\oplus$.

Thus, $\mathcal{T}$ can be considered as a basis of $(\mathcal{E}_n, \oplus, \otimes)$. This basis is unique. Nevertheless, there is no uniqueness of the decomposition of $A \in (\mathcal{E}_n, \oplus, \otimes)$ into the max-plus expression of matrices from $\mathcal{T}$.

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S05 - Poster

Commutative power-associative nilalgebras and Albert's problem

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Albert's problem ask if every commutative power-associative nilalgebra is solvable. We proof that commutative power-associative nilalgebras of dimension n and nilindex $n-3$ over a field algebraically closed of characteristic zero are solvable. Finally, we study commutative power-associative nilalgebras of dimension 9 and we proof that they are solvable too.

Joint work with Juan Carlos Gutierrez Fernandez (IME - USP).

S05 - Poster

A study on clean rings

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A ring is said to be clean if every element can be written as sum of a unit and an idempotent. These rings were defined by Nicholson [5], while studying exchange rings. The class of clean rings is located among other well known classes of rings [3]. In the realm of group rings, these properties have been studied from 2001 [2] on with the aim of characterizing the rings R and groups G such that the group ring RG is clean.

The study of $*$-clean rings was motivated by a question made by T. Y. Lam at the Conference on Algebra and Its Applications, in March 2005, at the Ohio University: which von Neumann algebras are clean as rings? Since von Neumann algebras are $*$-rings (i.e., rings with an involution), it is more natural to work with projections (idempotents that are symmetric under the ring involution) than with idempotents.

So, in 2010 Vaš defined $*$-clean rings [6]: a $*$-ring in which every element may be written as a sum of a unit and a projection. Clearly, every $*$-clean ring is a $*$-ring and is a clean ring.

Every group G is endowed with the classical involution $g \mapsto g^{-1}$. If R is a commutative ring, for instance, the R-linear extension of the classical involution in G is the classical involution in RG. $*$-clean group rings were first studied in 2011 [4]. However very little is still known about when a group ring is $*$-clean (not even the case of the group ring RG, where R is a commutative ring and G is a cyclic group, is fully stablished!).

In this talk, we present clean rings, their relationship with other types of rings [3] and some recent results [1]. Let R be a commutative local ring. I will provide necessary and sufficient conditions for the group rings RC_3 and RC_4 to be $*$-clean, where C_n denote the cyclic group with n elements.

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