# XXI CLA - Session S08
# Lie Groups and Representations

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S08 - July 28, 15:00 – 15:40

## 2-representations of Soergel bimodules

**Volodymyr Mazorchuk**
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The aim of this talk is to describe recent progress in the study of 2-representations of the 2-category of Soergel bimodules over the coinvariant algebra of a finite Coxeter group. For finite Weyl groups this 2-category is biequivalent to the 2-category of projective functors on the principal block of the BGG category O associated with the corresponding finite dimensional simple complex Lie algebra. In many cases, it turns out that simple transitive 2-representations of the 2-category of Soergel bimodules have Lie-theoretic interpretation, which we will try to explain. Finally, we will also explain an ADE-type classification of certain integral matrices which poped up in the study of Soergel bimodules for general dihedral groups.

*Joint work with Tobias Kildetoft (Uppsala University), Marco Mackaay (University of Algarve) and Jakob Zimmermann (Uppsala University).*

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S08 - July 28, 15:40 – 16:20

## Representation ring of Levi subgroups versus cohomology ring of flag varieties

**Shrawan Kumar**
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Recall the classical result that the cup product structure constants for the singular cohomology with integral coefficients of the Grassmannian of $r$-planes coincide with the Littlewood-Richardson tensor product structure constants for $GL(r)$. Specifically, the result asserts that there is an explicit ring homomorphism $\phi : \mathrm{Rep}_{poly}(GL(r)) \to H^*(Gr(r,n))$, where $Gr(r,n)$ denotes the Grassmannian of $r$-planes in $\mathbb{C}^n$ and $\mathrm{Rep}_{poly}(GL(r))$ denotes the polynomial representation ring of $GL(r)$.

This work seeks to achieve one possible generalization of this classical result for $GL(r)$ and the Grassmannian $Gr(r,n)$ to the Levi subgroups of any reductive group $G$ and the corresponding flag varieties.

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S08 - July 28, 16:20 – 17:00

## On the Demazure Type Structure of Graded Limits of Representations of Quantum Affine Algebras

**Adriano Moura**
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The finite-dimensional representation theory of quantum affine algebras has been subject of intense study for the past two decades motivated originally by the mathematical-physics literature. Although the irreducible representations have been classified in the early days of the development of the theory, unraveling their structure in general remains a chalenging problem. Recently, the character of several important classes of irreducible modules have been computed by relating them to Demazure modules. We shall discuss recent results in this direction. In particular, we present a result showing that Demazure modules of level 2 appear as the graded limits of representations in the subcategories introduced by Hernandez-Leclerc in connection to monoidal categorification of certain cluster algebras.

*Joint work with Matheus Brito (UC Riverside) and Vyjaynthi Chari (UC Riverside).*

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S08 - July 28, 17:30 – 18:10

## Positivity of parabolic Kazhdan-Lusztig polynomials

**Nicolas Libedinsky**
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We give diagrammatic categorifications of the spherical and of the anti-spherical modules for any Coxeter group. Our main theorem gives a "light leaves" basis of morphisms in these categorifications. We deduce that all flavours of parabolic Kazhdan-Lusztig polynomials have positive coefficients (for arbitrary choices of subsets of simple reflections).

*Joint work with Geordie Williamson (Max Planck Institut).*

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S08 - July 28, 18:10 – 18:30

## Tensor products of minimal affinizations in type $A$

**Fernanda Pereira**
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For a quantum affine algebra of type $A$, we describe the irreducible factors of the tensor product of a general minimal affinization with a Kirillov-Reshetikhin module associated to an extreme node of the Dynkin diagram of the underlying simple Lie algebra. More precisely, we give conditions on the Drinfeld polynomials for the tensor product of the corresponding irreducible modules to be irreducible. In the reducible case we show that the product has exactly two factors and describe them.

*Joint work with Adriano Moura (Universidade Estadual de Campinas, Brazil) and David Hernandez (Université Paris-Diderot Paris 7, France).*

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S08 - July 28, 18:30 – 18:50

## K-groups in the theory of symmetric spaces

**Wend Werner**
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The structure of symmetric spaces is very well encoded into an algebraic structure closely related to C*-algebras. Using this relationship to define K-theory for (hermitian, non-compact) symmetric spaces permits to replace root systems by K-groups in their classification. Classification beyond well-known results exist for inductive limits.

*Joint work with Dennis Bohle (Amsterdam).*

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S08 - July 29, 15:00 – 15:40

## Belavin-Drinfeld Lie bialgebras and quantum groups (Galois cohomology considerations)

**Arturo Pianzola**
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In the study of Lie bialgebra structures over C[[t]] certain "cohomology theories" were introduced by B. Kadets, E. Karolinsky, I Pop and A. Stolin. We will explain how these theories can be explained/reformulated in terms of Galois cohomology. By doing so we will be able to establish some open conjectures.

*Joint work with A. Stolin (Gothenburg, Sweden).*

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S08 - July 29, 15:40 – 16:20

## cohomology of finite, algebraic, and quantum groups

**Leonard Scott**
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I will discuss some interrelated topics in the cohomology and Ext groups for finite groups of Lie type, algebraic groups, and quantum groups. Of interest are 1)the negative solution to an old (1961) conjecture on maximal subgroups of finite groups, via counterexamples due to Frank Luebeck and a student of mine, Tim Sprowl, using Kazhdan-Lusztig polynomials and algebraic groups cohomology. 2)three conjectures by myself and Brian Parshall, on the interrelationship of Kazhdan-Lusztig polynomials and cohomology/Ext groups for modules in the algebraic groups case which come from irreducible modules for quantum groups. 3) the solution of the third of the above conjectures by a student, Hankyung Ko, of Parshall. One consequence is the calculation of all $\mathrm{Ext}^n$ groups between irreducible modules for quantum groups in type A at a root of unity, even with singular highest weights.

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## Gröbner bases for local Weyl modules for generalized current $\mathfrak{sl}_2$-algebras

**Angelo Bianchi**
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We use the theory of Gröbner bases for ideals to construct linear bases for the local Weyl modules for a generalized current algebra $\mathfrak{sl}_2 \otimes_{\mathbb{C}} \mathbb{C}[t_1, \ldots, t_n]$ associated to the finite-dimensional complex simple Lie algebra $\mathfrak{sl}_2$ and the polynomial algebra $\mathbb{C}[t_1, \ldots, t_n]$ with $n = 1, 2, 3$.

The main result is an explicit construction of linear bases for these important families of modules. In particular, we obtain some formulas to express the dimension of such modules. It is related to some works of Chari-Loktev, Chari-Pressley, Feigin-Loktev, and Loktev.

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S08 - July 29, 18:30 – 18:50

## Lie subalgebras of the matrix quantum pseudo differential operators

**Karina Batistelli**
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We give a complete description of the anti-involutions that preserve the principal gradation of the algebra $S_{q,N}$ of $N \times N$ matrix quantum pseudodifferential operators and we describe the Lie subalgebras of its minus fixed points. We obtain, up to conjugation, two families of anti-involutions that show quite different results when $n = N$ and $n < N$. Finally, we give a geometric realization of each of these anti-involutions and show their corresponding subalgebras are of classical type.

## References

V. G. Kac and A. Radul, *Quasifinite highest weight modules over the Lie algebra of differential operators on the circle*, Comm. Math. Phys. **157** (1993), 429–457.

V. G. Kac, W. Wang and C. Yan, *Quasifinite representations of classical Lie subalgebras of $W_{1+\infty}$* Adv. Math. **139** (1998), 56–140.

C. Boyallian, V. Kac, J. Liberati and C. Yan, *Quasifinite highest weight modules over the Lie algebra of matrix differential operators on the circle*, Journal of Math. Phys. **39** (1998), 2910–2928.

C. Boyallian and J. Liberati *Classical Lie subalgebras of the Lie algebra of matrix differential operators on the circle*, Journal of Math. Phys. **42** (2001), 3735-3753.

*Joint work with Carina Boyallian (CIEM-Famaf).*

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S08 - Poster

### Equidimensionality of some Gelfand-Tsetlin varieties

**Germán Benitez Monsalve**
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S. Ovsienko proved in 2003 that the Gelfand-Tsetlin variety for $gl(n)$ is equidimensional, i.e., all its irreducible components had the same dimension, in that case, such dimension is the dimension of affine space minus the number of equations. This result allows:

1. It guarantees the existence of irreducible modules in $gl(n)$ which are parameterized by the maximal espectrum of the Gelfand-Tsetlin subalgebra for $gl(n)$.

2. The universal enveloping algebra of $gl(n)$ is free as left and right module over its Gelfand-Tsetlin subalgebra.

In this poster, we will show the Gelfand-Tsetlin variety for $gl(n)$, the version for the quantum group Restricted Yangian of gl(n) and its equidimesionality.

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S08 - Poster

### $G_2$ holonomy manifolds are superconformal.

**Lázaro Orlando Rodríguez Díaz**
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We discuss the chiral the Rham complex (CDR) over a manifold $M$ with holonomy $G_2$. We will show how the vertex algebra of global sections of the CDR associated to $M$ contains two commuting copies of the Shatashvili-Vafa $G_2$ superconformal algebra.

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S08 - Poster

## Structure and Representations of Diferenttial Operators on the Torus

**João Schwarz**
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We discuss a noncommutative version of Noether's Problem to the ring of differential opertors on the torus, in the case the finite group is the symmetric group or a Weyl group of the families B and D. We also discuss some facts about simple weight modules of the invariants of the differential operator ring under the action of such groups.

*Joint work with Vyacheslav Futorny, (USP, Brasil).*

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S08 - Poster

## Chiral De Rham Complex Structure for Witt algebras

**André Eduardo Zaidan**
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The Chiral de Rham complex in the case of a torus $\mathbb{T}^N$, is a tensor product of two vertex super algebras: $V^+_{Hyp} \otimes V_{\mathbb{Z}^N}$, one is the hyperbolic latice vertex algebra and the other is the euclidean latice vertex algebra. The space $M_{Hyp}(\gamma) \otimes V^k_{\mathbb{Z}^N}$ has a structure of a module for the Witt algebra, , where $M_{Hyp}(\gamma)$ is a module for the hyperbolic latice vertex algebra and $V^k_{\mathbb{Z}^N}$ is the subspace of fermionic degree $k$. These modules exhaust all exceptional generalized highest weight modules for this Lie algebra.

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