XXI CLA - Session S09
Logic and Universal Algebra

S09 - July 25, 15:00 – 15:50

Monadic Gödel algebras are functional

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Monadic Heyting algebras where introduced by Monteiro and Varsavsky (1957) as an algebraic counterpart of the one variable fragment of quantified monadic intuitionistic logic, generalizing monadic Boolean algebras introduced by Halmos (1955) with a similar purpose for classical logic. They were found to interpret also the intuitionistic analogue of the modal system $S5$ and have been extensively studied since. Answering a question put by Monteiro, Bezhanishvili and Harding (2002) proved that any monadic Heyting algebra is functional; that is, it may be embedded in an algebra of functions $(H^X, \triangle, \nabla)$, where H is a complete Boolean algebra, the Heyting operations are defined pointwise, and the monadic operators $\triangle$ and ∇ are interpreted as $\triangle f = \inf_{x \in X} f(x)$, $\nabla f = \sup_{x \in X} f(x)$, respectively. Apart from the variety of boolean algebras for which Halmos proved a similar result, the situation is unknown for other familiar varieties of Heyting algebras. We solve this problem for monadic Gödel algebras, which interpret the one variable fragment of monadic predicate logic with values in the standard Gödel chain [0,1] (or in all linear Heyting algebras, Horn, 1969; Baaz et al, 2007). These are the monadic Heyting algebras satisfying the prelinearity axiom and the identity $\triangle(\triangle a \vee b) = \triangle a \vee \triangle b$ corresponding to the quantifier shift law $\forall x(\forall x\varphi \vee \psi) \leftrightarrow \forall x\varphi \vee \forall x\psi$, and constitute also the algebraic semantics of the Gödel analogue of S5 (Hájek, 2010, C. & Rodríguez 2012). Any monadic Gödel algebra may be embedded in an algebra of functions H^X where H is a complete Gödel algebra. For countable algebras, H may be chosen to be $[0, 1]$.

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Model theoretic properties of pseudo real closed fields.

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In this talk we will study the class of pseudo real closed fields (PRC-fields) from a model theoretical point of view. PRC-fields are a generalization of real closed fields and pseudo algebraically closed fields, where we admit having several orders. Prestel showed that the theory of PRC-fields can be axiomatized in the language of rings. We will explain some of the principal model-theoretic results of this theory, for example the form of the definable sets, the model-theoretic definable and algebraic closure, amalgamation of types and some new results obtained with Alf Onshuus and Pierre Simon about the definable groups.

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A new setting for duals of canonical extensions of lattices

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We consider properties of the graphs that arise as duals of bounded lattices in Ploscica's representation via maximal partial maps into the two-element set. We introduce TiRS graphs which abstract those duals of bounded lattices. We demonstrate their one-to-one correspondence with so–called TiRS frames which are a subclass of the class of RS frames introduced by Gehrke to represent perfect lattices. This yields a dual representation of finite lattices via finite TiRS frames, or equivalently finite TiRS graphs, which generalises the well-known Birkhoff dual representation of finite distributive lattices via finite posets. By using both Ploscica's and Gehrke's representations in tandem we present a new construction of the canonical extension of a bounded lattice. We present two open problems that can be of interest to researchers working in this area.

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MV-álgebras separables.

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Recordaremos la definición de objeto SEPARABLE en una categoría COEXTENSIVA, demostraremos que la categoría de MV-álgebras es coextensiva y caracterizaremos sus objetos separables. Además, enfatizaremos la analogía con el caso de anillos y explicaremos la naturaleza geométrica de los resultados.

Joint work with Vicenzo Marra (Università degli Studi di Milano).

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Fuzzy Neighborhood Semantics

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Our starting point is that the framework of classical logic is not enough to reason with vague concepts or with modal notions such as belief, uncertainty, knowledge, obligations, time, etc. Many-valued logical systems under the umbrella of mathematical fuzzy logic (in the sense of Hájek [1]) appear as a suitable logical framework to formalize reasoning with vague or gradual predicates, while a variety of modal logics address the logical formalization to reason about different notions as the ones mentioned above. Therefore, if one is interested in a logical account of both vagueness and some sorts of modality, one is led to study systems of many-valued modal logic.

The basic idea of this presentation is to systematically introduce modal extensions of many-valued or fuzzy logics. These logics, under different forms and contexts, have appeared in the literature for different reasoning modeling purposes. For instance, in [2], Fitting introduces a modal logic on logics valued on finite Heyting algebras, and provides a satisfactory justification to study such modal systems to deal with opinions of experts with a dominance relationship among them. In [3] and [4], the authors have proposed to extend Gödel fuzzy logic with modal operators. They provide a systematic study of this Gödal modal logic, which has been completmented in [5]. In [6], a detailed description of many-valued modal logics (with a necessity operator) over finite residuated lattice is proposed. In [7], a modal extension of Lukasiewicz logic is developed following an algebraic approach. Finally, in [8], a general approach to modal expansions of t-norm based logics is also introduced with the help of rational constants and possibly infinitary inference rules.

In most of these mentioned papers, many-valued modal logics are endowed with a Kripke-style semantics, generalizing the classical one, where propositions at each possible world, and possibly accessibility relations between worlds as well, are valued in a residuated lattice. The natural next step in this line of research is to axiomatize such semantics. However, this has turned apparently to be a considerable overall challenge because it is difficult to transfer some usual techniques from Boolean algebras to residuated lattices. For instance, the K axiom $(\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi))$ plays a central role in the construction of the canonical models in order to prove completeness in the classical case. However, except for either Gödel modal logic or many-valued modal logics defined from Kripke frames with crisp accessibility relations, the K axiom is not sound.

In order to overcome this difficulty, we propose to study an alternative semantics which is a generalization of the classical *neighborhood semantics*. This will be elaborated based on two preliminary workshop papers by the same authors ([9] and [10]). At this moment, it is worth mentioning some works from other authors which consider a generalization of neighborhood semantics in the same way we have done it. Namely, Kroupa and Teheux consider in [11] a neighborhood semantics for playable L_n-valued effectivity function. They want to characterize the notion of coalitional effectivity within game form models. Also we must mention a very recent paper by Cintula et al. ([12]) where the authors explore a fuzzified version of the classical neighborhood semantics and prove a relationship between fuzzy Kripke and neighborhood semantics in a very precise way (much better than the one proposed in our previous work). In fact, the authors of this paper propose to attack the problem of characterizing the modal extensions of MTL logics under a neighborhood semantics with algebraic tools. According to their algebraic approach, they characterize a global MTL modal logic, leaving open the case of characterizing the local consequence relation.

In summary, in this presentation, we will mainly focus on the development of a theoretical and general framework. Considering our motivation, our main goal, at large, is a systematic presentation of the minimum many-valued modal logics and their extensions. In this sense, we will firstly present minimum many-valued modal logics with necessity and possibility operators, $\Box, \Diamond$, defined on top of logics of residuated lattices under a neighborhood semantics.

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Joint work with Lluis Godo (IIIA-CSIC, Spain).

S09 - July 26, 15:00 – 15:50

MV-álgebras monádicas y l-grupos monádicos

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En esta charla presentamos una definición de los l-grupos monádicos con unidad fuerte y algunas propiedades básicas de esta clase de álgebras. Luego mostramos una equivalencia entre la categoría de los l-grupos monádicos y la categoría de las MV-álgebras monádicas que extiende a la equivalencia dada por D. Mundici con el funtor ?. Estudiamos las congruencias de un l-grupo monádico y las caracterizamos por medio de ciertos l-ideales monádicos. Probamos que el retículo de l-ideales monádicos es isomorfo al retículo de l-ideales de E(G). A partir de esta caracterización de las congruencias mostramos que todo l-grupo monádico es producto subdirecto de una familia de l-grupos monádicos G_i donde E(G_i) es una cadena para todo i. Para finalizar damos algunas aplicaciones de la equivalencia demostrada y comentaremos algunos de los avances en el estudio de los l-grupos monádicos como variedad .

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On hemi-implicative semilattices

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In this talk we introduce and study classes of algebras that properly include varieties of interest for logic. These algebras are obtained by weakening the main features of Heyting algebras but retaining most of their algebraic consequences. To be more precise, we give the following definition.

Definition: An algebra $(H, \wedge, \rightarrow, 1)$ of type $(2, 2, 0)$ is a *hemi-implicative semilattice* if the following conditions hold:

$H1$: $(H, \wedge, 1)$ is a bounded semilattice.

$H2$: For every $a, b, c \in H$, if $c \leq a \rightarrow b$ then $a \wedge c \leq b$.

$a \to a = 1$ for every $a \in H$.

An algebra $(H, \wedge, \vee, \to, 0, 1)$ of type $(2, 2, 2, 0, 0)$ is said to be a *hemi-implicative lattice* if $(H, \wedge, \vee, 0, 1)$ is a bounded distributive lattice and $(H, \wedge, \to, 1)$ is a hemi-implicative semilattice.

If $(H, \wedge)$ is a semilattice with a binary operation $\to$, then H satisfies the condition (H2) if and only if it holds the inequality $a \wedge (a \to b) \leq b$ for every $a, b \in H$. Thus, the condition (H2) is a kind of modus ponens rule. Moreover, the class of hemi-implicative semilattices is a variety and the class of hemi-implicative lattices is also a variety.

Implicative semilattices introduced by Nemitz are examples of hemi-implicative semilattices. Some examples of hemi-implicative lattices are the semi-Heyting algebras, which were introduced by H.P. Sankappanavar in as an abstraction of Heyting algebras, and the RWH-algebras, which were introduced by Celani and Jansana in as another possible generalization of Heyting algebras.

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Epic subalgebras and primitive functions

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Stonean residuated lattices

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By a Stonean residuated lattice I mean a bounded integral residuated lattice-ordered commutative monoid satisfying the equation

$$\neg x \vee \neg\neg x = \top.$$

I will show that stonean residuated lattices are characterized by triples $\langle \mathbf{B}, \mathbf{D}, \varphi \rangle$ where $\mathbf{B}$ is a Boolean algebra, $\mathbf{D}$ is an unbounded residuated lattice and φ is an order reversing homomorphism from $\mathbf{B}$ into the lattice of implicative filters of $\mathbf{D}$.

Joint work with Manuela Busaniche (Universidad Nacional del Litoral, Argentina).

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An optimal axiomatization of the set of central elements

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We say that a variety $\mathcal{V}$ with $\vec{0}$ and $\vec{1}$ has "definable factor congruences" if there exists a first-order formula defining every factor congruence in every algebra $\mathbf{A} \in \mathcal{V}$ in terms of its associated *central elements*. When there is a $(\bigwedge p = q)$-formula satisfying this condition we say that $\mathcal{V}$ has "equationally definable factor congruences". We denote by $Z(\mathbf{A})$ the set of central elements of $\mathbf{A}$. In "Varieties with equationally definable factor congruences II" we give an axiomatization of $Z(\mathbf{A})$ for varieties with equationally definable factor congruences which is optimal in the sense of its quantificational complexity. The given axiomatization is not a set of positive formulas nor a set of Horn formulas. There are several examples which show that in the general case, varieties with equationally definable factor congruences do not admit an axiomatization of $Z(\mathbf{A})$ by a set of positive formulas. However, as we will see, there is an axiomatization of $Z(\mathbf{A})$ which is a set of Horn formulas with the optimal quatificational complexity, which evidences the already known fact that central elements are preserved by direct products.

Joint work with Diego Vaggione (Universidad Nacional de Córdoba).

S09 - Poster

Límites inversos de estructuras compactas

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Una estructura de primer orden A se dice atómicamente compacta si cualquier conjunto de fórmulas atómicas con constantes en A que sea finitamente satisfactible en A, es satisfactible en A. Si A es un álgebra las atómicas son igualdades de polinomios. En [2] se prueba que esta noción es equivalente a que A sea retracto de cualquier extensión pura. Es fácil ver que si A es compacta, es decir, tiene una topología compacta de Hausdorff que hace a las operaciones continuas y a las relaciones cerradas, entonces A es atómicamente compacta. Por lo tanto cualquier límite inverso de estructuras compactas es atómicamente compacto. En [1] se muestra que cualquier límite inverso de un sistema inversamente dirigido de estructuras finitas es retracto de un ultraproducto en estas estructuras. En el poster que proponemos se muestra que el mapa natural del límite inverso de un sistema de estructuras compactas en un ultraproducto de estas estructuras es puro. Esto sumado a que el límite inverso es atómicamente compacto da una prueba de un resultado mas general que el de [1]

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Joint work with Xavier Caicedo (Universidad de los Andes, Colombia).

S09 - Poster

The ring $\prod_{n=1}^{\infty} F_{p_i}$

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This poster presents the structure of the ring $\prod_{n=1}^{\infty} F_{p_i}$, where p_i is the i^{th} prime, $p_1 = 2; p_2 = 3; \ldots,$ and details a relationship of principal ideals within the ring with subsets of the natural numbers.

We try to understand the ring by determining if it is finitely generated, a Von Neumman regular ring, and the relationship with the weak direct product. We examine first order definable sets in this ring and attempt to topologyze it using dictionary order. Also we present the elements with torsion and cyclotomic polynomials.

Joint work with Sergio Palomo (City University of New York).
