XXI CLA - Session S10
Homological Methods

S10 - July 28, 15:00 – 15:30

Nearest Points on Toric Varieties

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We determine the Euclidean distance degree of a projective toric variety. This extends the formula of Matsui and Takeuchi for the degree of the A-discriminant in terms of Euler obstructions. The primary goal of the project is the development of reliable algorithmic tools for computing the points on a real toric variety that are closest to a given data point. In this lecture we emphasise the role played by characteristic classes such as the Chern-Mather class.

Joint work with Martin Helmer (UC Berkeley, USA).

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Codimension two A-hypergeometric systems

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A-hypergeometric systems are systems of partial differential equations associated to toric ideals, introduced by Gelfand, Graev, Kapranov and Zelevinsky in the late 1980s. Homological methods have proved exceedingly effective in studying hypergeometric equations, and provide the only known way of computing the dimension of the solution space of a hypergeometric system. However, homological considerations have so far failed to explain how the solutions themselves behave.

I will explain an approach towards bridging this gap in the case of codimension two lattice ideals. Using the combinatorial minimal free resolutions of codimension two lattice ideals constructed by Peeva and Sturmfels, I will present Ext and local cohomology computations, which, taken together, control the behavior of the corresponding A-hypergeometric functions. Along the way, we give an explicit combinatorial formula for the graded local duality isomorphisms in this case.

Joint work with Roberto Barrera (Texas A&M University, USA) and Christine Berkesch Zamaere (University of Minnesota, USA).

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On Unimodality of Hilbert Function Graded Algebras

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We will survey the problem of determining Unimodality of Hilbert Functions especially for graded algebras of small codimensions. Let $R = \oplus R_n$ be a standard graded algebra over a field k an I be a homogeneous ideal so that $S = R/I = \oplus S_n$ is a graded algebra of dimension zero. Then the Hilbert function of R/I, denoted by $h_I(n) = h_S(n) = dim_k S_n$ is a function such that $h_S(0) = 1, h_S(1) = e$, the embedding dimension of S and $h_S(n) = 0$, for $n > s$, where s is the socle degree of S. Hilbert function is called unimodal if $h_0 \leq h_1 \leq \cdots \leq h_{t-1} \leq h_t \geq h_{t+1} \geq cdots \geq h_s \geq h_{s+1} = 0$ for some t. Hilbert functions of Gorenstein algebras are also symmetric. So, if they are unimodal, $t = s/2$ or $(s+1)/2$. It is known that Hilbert function of Goresntein algebras are unimodal in codimensio three and it is as yet open in codimension 4. There are examples of non unimodal Cohen Macaulay algebras codimension 3 and Gorenstein algebras in codimension 5 and higher. We will discuss the problem and some recent resutls in Codimension 3 level algebras and Gorenstein algebras of codimension 4.

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The syzygies of some thickenings of determinantal varieties

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The space of m x n matrices admits a natural action of the group GL_m x GL_n via row and column operations on the matrix entries. The invariant closed subsets are the determinantal varieties defined by the (reduced) ideals of minors of the generic matrix. The minimal free resolutions of these ideals are well-understood by work of Lascoux and others. There are however many more invariant ideals which are non-reduced, and they were classified by De Concini, Eisenbud and Procesi in the 80s. I will explain how to determine the syzygies of a large class of these ideals by employing a surprising connection with the representation theory of general linear Lie superalgebras.

Joint work with Jerzy Weyman (University of Connecticut).

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Title: Invariant theory of Milnor algebras

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Given a non-degenerate homogeneous form f on $\mathbb{C}^n$ of degree d, the Milnor algebra of f is defined as the quotient of the polynomial ring $\mathbb{C}[x_1, ..., x_n]$ by the ideal $J(f)$ of first order partials of f. For each integer k, one can define the kth Hilbert point of the Milnor algebra as the subspace of degree k polynomials contained in $J(f)$. When $k = n(d-2)$, this Hilbert point is classically called a Macaulay inverse system. We study the invariant theory of the these Hilbert points viewed as points in the corresponding Grassmanians. We will then be able to resolve a conjecture of Eastwood and Isaev which is related to the well-known Mather-Yau theorem for isolated hypersurface singularities.

Joint work with Alex Isaev (Australian National University) and Maksym Fedorchuk (Boston College).

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Relative Igusa-Todorov functions

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We develop the theory of the $\mathcal{E}$-relative Igusa-Todorov functions in an exact IT-context $(\mathcal{C}, \mathcal{E})$. In the case when $\mathcal{C} = \text{mod}\,(\Lambda)$ is the category of finitely generated left Λ-modules, for an artin algebra Λ, and $\mathcal{E}$ is the class of all exact sequences in $\mathcal{C}$, we recover the usual Igusa-Todorov functions. We use the setting of the exact structures and the Auslander-Solberg relative homological theory to generalise the original Igusa-Todorov's results. Furthermore, we introduce the $\mathcal{E}$-relative Igusa-Todorov dimension and also we obtain relationships with the relative global and relative finitistic dimensions and the Gorenstein homological dimensions.

Joint work with Octavio Mendoza (Universidad Nacional Autónoma de México).

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Applications of volume to commutative algebra

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We give general conditions under which volumes and volume like functions exist and some applications to problems in commutative algebra. We give some examples where volumes and volume like functions do not exist.

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Nonsymmetric operads, associative algebras, and the Lagrange inversion

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I shall discuss a construction of a functor from associative algebras to nonsymmetric operads which has good homological and homotopical properties. As a consequence, I shall give a new categorical context for the Lagrange inversion formula. Another consequence I shall mention concerns various examples and counterexamples in Koszul duality for operads.

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Hochschild homology and cohomology of down-up algebras

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We have computed Hochschild homology and cohomology of homogeneous down-up algebras in the generic case and in the Calabi-Yau case.

Let K be a fixed field. Given parameters $(\alpha, \beta, \gamma) \in K^3$, the associated down-up algebra $A(\alpha, \beta, \gamma)$ is defined as the quotient of the free associative algebra $K\langle u, d\rangle$ by the ideal generated by the relations

$$\begin{aligned} d^2u - (\alpha dud + \beta ud^2 + \gamma d), \\ du^2 - (\alpha udu + \beta u^2 d + \gamma u). \end{aligned} \tag{1}$$

This family of algebras was introduced by G. Benkart and T. Roby. As typical examples we have that $A(2, -1, 0)$ is isomorphic to the enveloping algebra of the Heisenberg-Lie algebra of dimension 3, and, for $\gamma \neq 0$, $A(2, -1, \gamma)$ is isomorphic to the enveloping algebra of $\mathfrak{sl}(2, K)$. Moreover, Benkart proved that any down-up algebra such that $(\alpha, \beta) \neq (0, 0)$ is isomorphic to one of Witten's 7-parameter deformations of $U(sl(2, K))$.

E. Kirkman, I. Musson and D. Passman proved that $A(\alpha, \beta, \gamma)$ is noetherian if and only it is a domain, which is tantamount to the fact that the subalgebra of $A(\alpha, \beta, \gamma)$ generated by ud and du is a polynomial algebra in two indeterminates, that in turn is equivalent to $\beta \neq 0$. Under either of the previous situations, $A(\alpha, \beta, \gamma)$ is Auslander regular and its global dimension is 3. On the other hand, it was proved by Cassidy and Shelton that, if K is algebraically closed, then the global dimension of $A(\alpha, \beta, \gamma)$ is always 3. Moreover, Benkart and Roby also proved that the Gelfand-Kirillov dimension of a down-up algebra is 3, independently of the parameters.

If $\gamma = 0$, the down-up algebra can be regarded as nonnegatively graded, where the degree of u and d is 1. In this case, the algebra is 3-Koszul and Artin-Schelter regular.

Joint work with Sergio Chouhy. IMAS-CONICET, Argentina and Estanislao Herscovich. Universidad de Buenos Aires and Institut Joseph Fourier, Grenoble, France.

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Homología cíclica de productos cruzados débiles (Cyclic homology of weak crossed products)

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Primero revisamos el concepto de productos cruzados débiles presentado en [1] y [2]. Este concepto generaliza al presentado en [3] y contiene al de productos cruzados de álgebras por álgebras de Hopf débiles. Luego estudiamos las homologías de Hochschild y cíclica de estos productos cruzados. En primer lugar construímos una resolución para los productos cruzados débiles que es más simple que la canónica y luego la usamos para obtener complejos que calculen su homología y cohomología de Hochschild respectivamente. Finalmente, usando el lema de perturbación, obtenemos un complejo mezclado que da las homologías cíclica, periódica y negativa de estas álgebras.

1) J. N. Alonso Álvarez, J. M. Fernández Vilaboa, R. González Rodríguez y A. B. Rodríguez Raposo, Crossed products in weak contexts, Appl. Categ. Structures 18 (2010), no. 3, 231-258.

2) J. M. Fernández Vilaboa, R. González Rodríguez y A. B. Rodríguez Raposo, Preunits and weak crossed products, Journal of Pure and Applied Algebra 213 (2009), 2244-2261.

3) Tomasz Brzezinski, Crossed products by a coalgebra, Comm. Algebra 25 (1997), no. 11, 3551-3575.

First we review the concept of weak crossed products introduced in [1] and [2]. This concept generalizes to that introduced in [3] by Brzeziński and it contains the crossed products of algebras by weak Hopf algebras. Then we study the Hochschild and cyclic homology of these crossed products. First, for the weak crossed products, we built a resolution that is simpler than the canonical one and then we use it for obtain complexes that calculate their Hochschild homology and cohomology respectively. Finally, using the perturbation lemma, we get a mixed complex that gives the cyclic, periodic and negative homologies of these algebras.

Joint work with Juan José Guccione (Universidad de Buenos Aires) y Christian Valqui (Pontificia Universidad Católica de Perú).

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Isomorphism conjectures with proper coefficients

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Let G be a group and $\mathcal{F}$ a nonempty family of subgroups of G, closed under conjugation and under subgroups. Also let E be a functor from small $\mathbb{Z}$-linear categories to spectra, and let A be a ring with a G-action. Under mild conditions on E and A one can define an equivariant homology theory $H^G(-, E(A))$ of G-simplicial sets such that $H^G_*(G/H, E(A)) = E(A \rtimes H)$. The strong isomorphism conjecture for the quadruple $(G, \mathcal{F}, E, A)$ asserts that if $X \to Y$ is an equivariant map such that $X^H \to Y^H$ is an equivalence for all $H \in \mathcal{F}$, then $H^G(X, E(A)) \to H^G(Y, E(A))$ is an equivalence. We introduce an algebraic notion of $(G, \mathcal{F})$-properness for G-rings, modelled on the analogous notion for G-C^*-algebras, and show that the strong $(G, \mathcal{F}, E, P)$ isomorphism conjecture for $(G, \mathcal{F})$-proper P is true in several cases of interest in the algebraic K-theory context.

Joint work with Guillermo Cortiñas (Universidad de Buenos Aires, Argentina).

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BV-algebra Structures and Hochschild (Co)homology

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In this talk we will recall the BV-algebra structures of the Hochschild cohomology of symmetric algebras and its relation with string topology. We will present the calculations of these structures for the group ring of finitely generated abelian groups. If time permits, we will present the notion of Hochschild Tate cohomology defined by Buchweitz and the BV-algebra structure defined by Wang for self-injective algebras. As an example, we will show the calculations for the group ring of cyclic groups.

Joint work with Andrés Angel (Universidad de los Andes, Colombia).
